AN EXPERIMENTAL ANALYSIS OF TRANSIENT VEHICLE LOADS AND RESPONSE OF FLEXIBLE PAVEMENTS

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by

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Joint Highway Research Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

Final Report

AN EXPERIMENTAL ANALYSIS OF TRANSIENT

VEHICLE LOADS AND RESPONSE OF FLEXIBLE PAVEMENTS

To: G. A. Leonards, Director
Joint Highway Research Project

September 24, 1965

From: H. L. Michael, Associate Director
Joint Highway Research Project

File No: 6-20-6
Project No: C-36-52F

The Final Report attached "An Experimental Analysis of Transient Vehicle Loads and Response of Flexible Pavements" is submitted on the HPR research project Stresses and Deflections. The report has been authored by Mr. John L. Sanborn of our staff under the direction of Professor E. J. Yoder. Mr. Sanborn also utilized the report as his Ph.D. dissertation.

The report describes the development and application of equipment and techniques for measuring simultaneously the dynamic loads applied to a pavement by a moving vehicle and the dynamic stresses and displacements induced by that load in a flexible pavement and its subgrade.

The report will be transmitted to the Indiana State Highway Commission and the Bureau of Public Roads for their review, comments and approval. It is reported to the Board for its acceptance.

This report together with the accompanying Final Report by G. W. Kibbee constitutes theFinal Report on the Stresses and Deflection HPR project. Additional acti ity on this research project may be proposed after all parties have reviewed the report and its suggestions for further research.

Respectfully submitted,

Harold L. Michael

Harold L. Michael, Secretary

HLM:bc

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Final Report

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by

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Joint Highway Research Project
File No: 6-20-6
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Lafayette, Indiana
September 24, 1965

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This study was supported by the Indiana State Highway Commission and the U. S. Bureau of Public Roads. It was administered by the Joint Highway Research Project under the direction of Professor K. B. Woods and Professor H. L. Michael. The State Highway Department lent invaluable assistance by providing a truck and operator with supporting personnel and equipment for running field tests.

The tire pressure measurements and tire force calculations were made by G. W. Kibbee under the supervision of Professor B. E. Quinn of the School of Mechanical Engineering. Mr. Kibbee adapted the pressure measuring equipment for heavy vehicles and dual tires, and developed the technique of interpreting the pressure record in terms of force with respect to vehicle position on the highway.

Especial gratitude is extended to Professor E. J. Yoder who served as the author's major professor. Professor Yoder provided the initial stimulus and continuing encouragement which made it possible to pursue this study. The freedom with which he permitted the author to pursue ideas and techniques and to grapple with interpretation is greatly appreciated.

Particular thanks is due also to Professor A. D. M. Lewis who provided invaluable assistance in developing equipment and in computer programming. Until his absence during the final stages of this thesis, Professor Lewis served on the author's committees.

The author acknowledges with thanks the support, advice and encouragement of these men, along with that of the many staff members and students of Purdue University who contributed much, directly and indirectly, to the conduct of the work.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

LIST OF ILLUSTRATIONS (Continued)

ABSTRACT

Sanborn, John Leonard. Ph.D., Purdue University, August 1965. An Experimental Analysis of Transient Vehicle Loads and Response of Flexible Pavements. Major Professor: E. J. Yoder.

Many procedures are in current use for designing and evaluating flexible pavements. All of these procedures rely to some degree on empirical relations of pavement performance and vehicle loads. None of them take into account the effect of vehicle velocity on the wheel load or the dynamic response of the pavement to that load. The evaluation of pavement response has involved measurement of stress and of displacement, but not both simultaneously. Furthermore, the load used in analysis of data has generally been simply the static wheel (or axle) load.

This report describes the development and application of equipment and techniques for measuring simultaneously the dynamic loads applied to a pavement by a moving vehicle, and the dynamic stresses and displacements induced by that load in a flexible pavement and its subgrade. Dynamic loads were measured by monitoring the fluctuations in tire inflation pressure. Dynamic calibration techniques were applied to obtain the force-pressure ratio for the tire-vehicle combination as a function of frequency. Field measurements of tire pressure were resolved into frequency components, transformed to force and recombined to obtain the total force exerted on the pavement.

Stresses were measured with a cell incorporating the flexible diaphragm principle. Displacements were sensed with an acceleration sensitive transducer. An electronic analog device was used to obtain the first integral of the acceleration signal, and a numerical integration of that result was used to obtain displacement.

Data were obtained from three pavements of differing stiffness constructed over similar subgrades. In each pavement, stress and displacement transducers, mounted together as a single unit, were placed at the surface-base interface, the base-subgrade interface, and in the subgrade three feet from the surface. A single axle dump truck, loaded to 9,000 pounds on each dual wheel, was operated over the test sections at speeds varying from 5 to 60 mph.

Plots of stress and displacement are presented in which an important dependence on lateral position of the test vehicle is apparent. Trends of stress and displacement as a function of vehicle velocity are also indicated. Statistical curve fitting techniques were used to fit the data to a model which relates peak stress and displacement to instantaneous load at the test point, lateral position of the vehicle and vehicle velocity. Certain results are presented which indicate that dynamic effects of load on pavement response were unimportant in the case of the vehicle and pavements in this study.

Using the fitted equations, predicted values of stress and displacement were obtained for particular values of load and vehicle position as a function of vehicle velocity. The ratios of average vertical stress to average vertical strain in the base course and in the subgrade were computed from predicted values of stress and displacement and the variation of these ratios with vehicle velocity is demonstrated.

1. INTRODUCTION

The establishment of adequate design techniques and criteria for highway pavements requires among other factors a knowledge of the loads to be carried and the response of the pavement to those loads. There are many procedures in current use for designing and evaluating pavements (50)*, all of them relying to some degree on empirical relationships of pavement performance and traffic loads. These methods in general are based on a nominal wheel (or axle) load derived from static weights, and the properties of pavement components as measured by tests which are either static or performed at low rates of strain. None of the methods account for the effect of vehicle velocity on the wheel load or the dynamic response of the pavement to that load.

It has been observed (50) that pavement distress is greatest when vehicles are slow moving or standing. Measurements of displacement (26, 28, 30) generally indicate greater pavement displacement at low vehicle speed or when the vehicle is standing than at high speeds, although these tests are generally not made at normal operating speeds for highways. On the other hand, tests of tire forces on pavements (39) indicate that the "average" force on the pavement produced by a moving vehicle increases with vehicle velocity. In addition, one series of tests of rigid pavement displacements (21) shows a decrease in displacement only for moderate vehicle velocities. The data from these tests suggest the possibility that displacements may increase at velocities greater than 30 to 40 mph.

* Numbers in parentheses refer to items listed in Bibliography.

Vehicle loads, pavement stresses and displacements have all been studied by various investigators. Measured displacements, in particular, have been used by many engineers in the evaluation of pavements. With the few exceptions already noted, measurements of stress and displacement have been made only for static or slow moving loads. Furthermore, stress and displacement measurements have not been made simultaneously. Studies of pavement stresses have compared field measurements to theoretical values and where analysis required stress-strain properties of the materials, these values have either been assumed or taken from laboratory tests. Displacement measurements also have been made by many investigators. However, analysis of field measurements which require a knowledge of stresses in the mass have invariably been made on the basis of theoretical stress values.

Dynamic loads of highway vehicles have been studied by comparatively few investigators. Those studies that have been reported in the literature in general consider the load independently of its effect on the pavement structure and subgrade.

In understanding the behavior of pavements in reacting to vehicle loads, it is essential that the factors mentioned above be considered simultaneously. Hveem and Carmany (33), in a classic statement on pavement design, pointed out the importance of all of these factors. Burmister (11, 12), Baker and Papazian (5), and others have pointed out the effects of relative stiffness of pavement materials and subgrade soil on response of a pavement to load. Casagrande and Shannon (15) and Hampton and Yoder (24) have noted that soil deformation characteristics and strength vary

with the rate at which load is applied. In considering a vehicle moving over a highway the question arises as to the amount and effect of impact loading, and whether tests using static or slowly moving loads give a realistic indication of pavement behavior under service conditions. Only with simultaneous measurements is it possible to make valid judgments of pavement characteristics and to learn the true effect of vehicle velocity and type of pavement.

For a number of years personnel in the Soils Laboratory of the Joint Highway Research Project at Purdue University have directed a considerable effort towards obtaining field measurement of stress-strain properties of pavements and of loads imposed on pavements. A pressure cell was developed by McMahon and Yoder (37) which gave satisfactory results in subgrades, at least for static loads. Walker, et al. (44) measured both surface and layer displacements of flexible pavements. He showed that the individual layer measurements are significant at least in relating pavement cracking to properties of a given layer. Wilson (48) developed suitable equipment and techniques for measuring dynamic tire forces and interpreting them in terms of total load on the pavement. These and other studies indicate that the simultaneous measurement of these factors will be of particular value in evaluating pavement performance and improving design criteria.

The study described in this report was carried out between the fall of 1962 and the summer of 1965. Development and calibration of equipment was done in the laboratories of the Joint Highway Research Project and the School of Mechanical Engineering at Purdue University. Subsequently, field tests were run on sections of three state highways in the vicinity of Lafayette, Indiana. The location of the test sites is indicated in Figure 1.1.

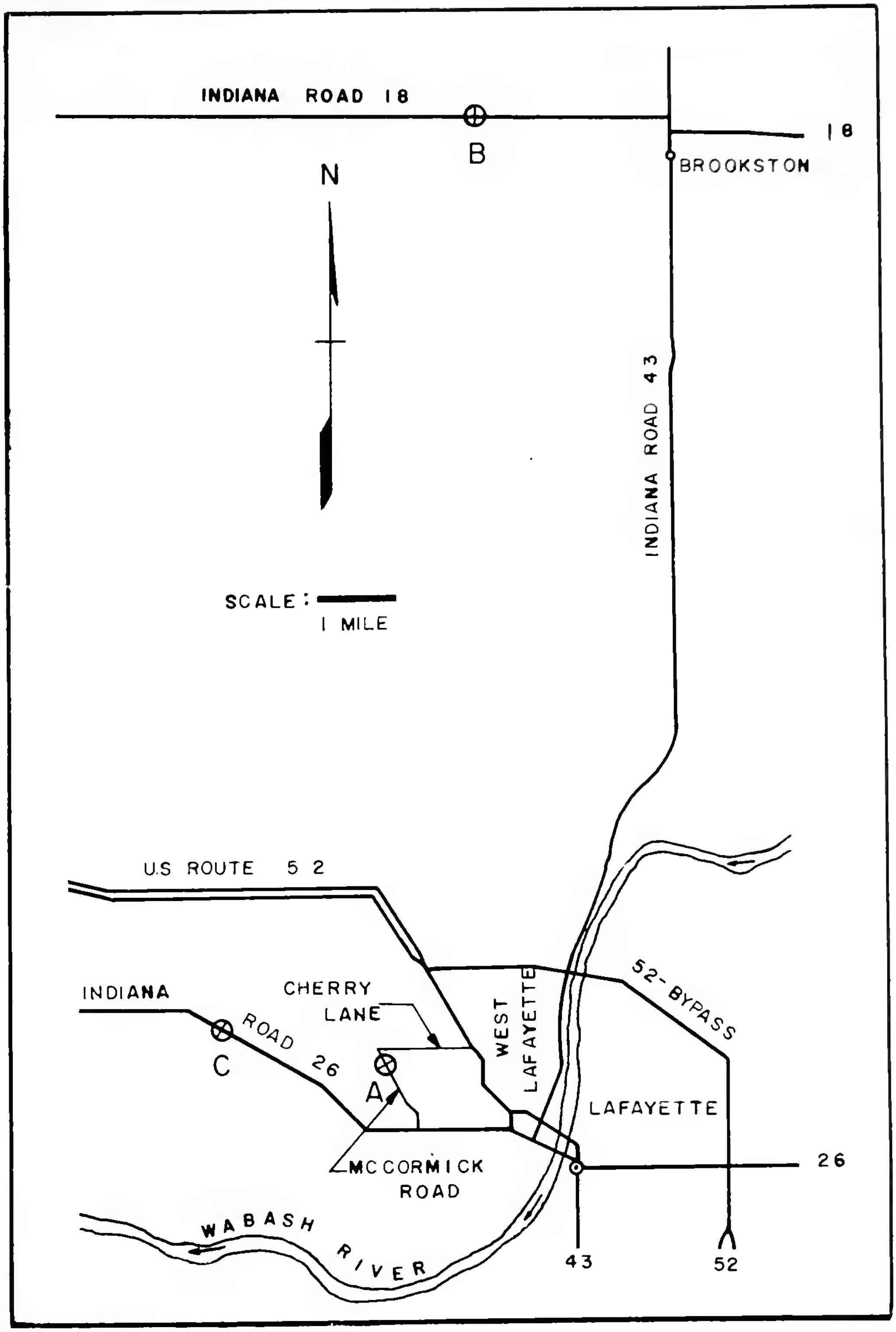


FIGURE 1.1. LOCATION OF TEST PAVEMENTS, 1964

2. PURPOSE AND SCOPE

This report summarizes the initial phases of a project related to transient response of flexible pavements to moving loads. The major objectives of the over-all project are to study, 1) the variation in tire load with vehicle velocity, 2) the effect of this change in load on pavement response, and 3) the dynamic response characteristics of pavement materials.

To accomplish the above, the primary purposes of this study were to, 1) further the development of equipment and techniques for measuring flexible pavement stresses and displacements and the applied dynamic loads under transient conditions, and 2) study items 1 and 2 of the over-all objectives. Item 3 above was studied but time limitations did not permit this in detail.

It was desired to obtain measurements of stress and displacement not only at the pavement surface, but also at discrete points within the layered system and in the subgrade. The data were to be obtained on actual flexible pavements of varying stiffness using a conventional heavy highway vehicle for the test load. Furthermore, the measurements were to be made over a wide range of vehicle velocities, varying preferably from static to greater than 60 mph. Equipment and techniques were to be so designed that a continuous and simultaneous record of all parameters would be obtained for all test conditions.

It was anticipated that the study would be accomplished in several distinct phases. Building on the foundation of previous projects, the Soils Laboratory was to undertake the development of stress and displacement measurements. Working separately, but in coordination with the Soils Laboratory, personnel from the School of Mechanical Engineering were to further the development of the load measuring equipment, including the adaptation to heavy vehicles and suitable calibration techniques. Both of these phases would require considerable laboratory investigation to evaluate equipment and techniques as well as to calibrate the systems. After suitable equipment was developed, it was intended to run field tests to meet the primary objectives -- that is, the actual simultaneous measurement of pavement stress and displacement and the corresponding applied load.

With the accomplishment of physical measurements, analysis of the data would be made in such a manner as to evaluate the variation of applied loads, and to determine the extent and character of dependence of this variation on the speed of the test vehicle and pavement construction. No attempt was planned to develop a rigorous mathematical analysis of the problem which would involve the addition of the time domain to the elastic analyses which have already been published (3, 7, 11, 40). Measurements to be made in this study were restricted to vertical forces, stresses and displacements. Also, tests were made for only one nominal axle load. The effects of tractive forces and of variation of static loads, though probably of importance (6, 33, 46) to the total problem were considered beyond the scope of this work.

3. THE DYNAMIC LOAD MEASURING SYSTEM

A variety of techniques have been devised for measuring dynamic forces exerted by the wheel of a moving vehicle on a pavement. These methods include measurements of the strain or acceleration of the axle housing, deflections of the tire sidewalls, bending of specially constructed rims and measurements of tire inflation pressure. Wilson (48) has pointed out that of these, the last method has a number of important advantages over the others. In this case, the parameter being observed is physically closest to the force which is to be measured. This assures a minimum of inertial effect between actual tire force and the indicated record of force. In addition, the system is easily and quickly mounted on (or removed from) the test vehicle and it is extremely sensitive. It was felt that these advantages overcame potential difficulties in calibration and of heating and cooling effects on tire pressure which would have to be accounted for in order to make reliable mesurements of dynamic tire force.

Measurement of Tire Pressure Variations

Early methods of measuring dynamic tire pressures are reported by Boswell and Hopkins (10) in which a pressure transducer was mounted directly on the wheel. The electrical signal from the transducer was transmitted through sliprings to the recording equipment. This system was improved significantly by other researchers (22, 30) by introducing a rotating pressure seal through which the pressure would be transmitted from the rotating tire to a differential pressure transducer mounted on

the body of the vehicle. Wilson (48) in 1964 described in detail the development of such a system for use on a passenger vehicle. The equipment is illustrated schematically in Figure 3.1.

In the diagram of Figure 3.1, the elements shown on the instrument board are fixed to the body of the vehicle. Pressure from the rotating tire is transmitted through the special rotating joint on the wheel and through flexible tubing to side A of the differential pressure transducer. Side B of the transducer is exposed to pressure from the reference tank. At the beginning of a test, pressures in the tire and reference tank are equalized by opening valves No. 1 and No. 2. Valve No. 1 is then closed, while valve No. 2 remains open for the duration of the test. Fluctuations in the tire pressure above and below the initial, or reference, pressure are then sensed by pressure changes in side A of the transducer relative to the constant pressure on side B. An electric signal produced by the transducer is transmitted to electronic equipment for amplification and recording on an oscillograph record. The function of the by-pass is to account for slow changes of pressure in the tire which result from leakage or temperature changes as opposed to variations in load.

The first attempt to account for leakage and temperature variations incorporated a solenoid operated valve in the by-pass. This device could be actuated by the observer as the vehicle was in motion, thus permitting the equalization of the tire and reference pressures at any time in the course of a test run. This capability permits the running of tests without long operation of the test vehicle ahead of time to establish thermal equilibrium in the tires. A serious disadvantage of intermittent equalization became apparent in early road tests, however. If activation of

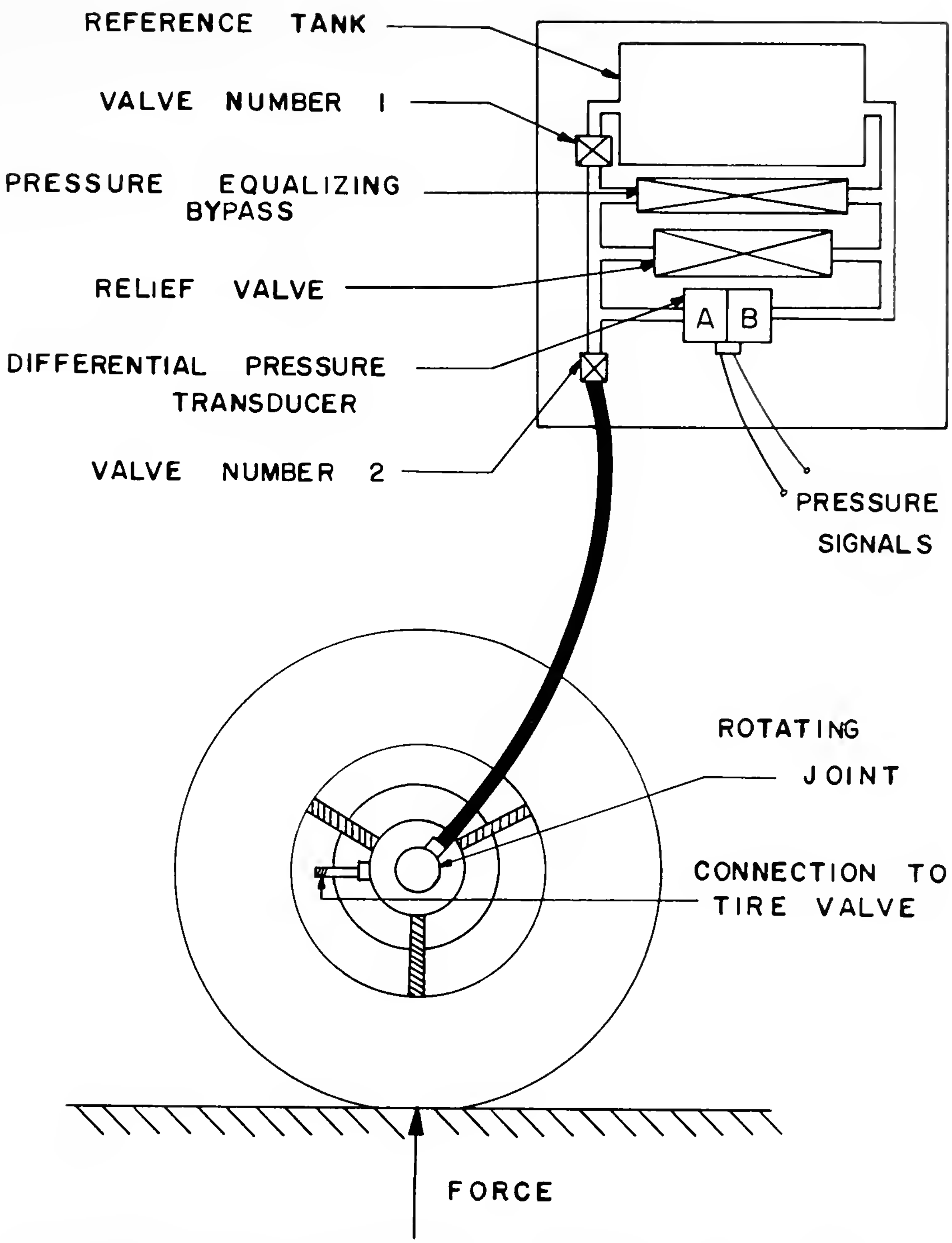


FIGURE 3.1. SCHEMATIC DIAGRAM OF TIRE PRESSURE MEASURING SYSTEM (AFTER WILSON, PH. D. THESIS, PURDUE UNIVERSITY, 1964)

the spring-loaded valve should result in its closing at an instant when the tire pressure is not equal to the static pressure, the resulting pressure variation on the oscillograph record would be about a base different from the static pressure. In addition, during a long test heating effects on the tire pressure may be so great as to require that the valve be actuated during the course of a test, resulting in a discontinuity in the base line.

To overcome the above problems, a method of continuous equalization was employed in which the solenoid valve was replaced by a small orifice. The size of the restriction was carefully chosen to insure that the equalization rate would be slow enough so that dynamic variations due to vehicle operation would not be affected, yet a slowly changing pressure such as produced by heating of the tire or leakage would be continuously counteracted. The major shortcoming of this system is that it prevents static calibration. However, dynamic calibration proved desirable for other reasons as well, so that the continuous equalization could be adopted satisfactorily.

Wilson's equipment built for a passenger vehicle was readily adaptable to heavy trucks for monitoring the force variation on a single tire, or even that on a dual wheel if the tires were interconnected to maintain the same pressure in both. The major differences were the higher pressure of the truck tires (about 80 psi compared to 20 to 30 psi for the automobile), and the tendency to decrease the natural frequency of the system because of the increased length of tubing necessary between the rotating joint and the instrument panel.

In order to obtain realistic measurements, however, it was necessary to develop a means of monitoring separately pressures of independent dual wheels. The solution of this problem, as well as minor adaptation problems, and further development of calibration and interpretation techniques was undertaken by G. W. Kibbee and Professor B. E. Quinn. The details of that study are to be reported separately. No commercially produced rotating seal was available which was capable of transmitting two separate pressures. To meet this need, two Aeroquip Corp. Barco Rotary Joints, Type E, were obtained and modified so as to enable mounting them on a single axis. The entire unit was assembled on a flange and bolted to the outer wheel of the dual pair. In addition, a new instrument panel was prepared with two Statham pressure transducers, one PM295TC and one PM722TC, along with reference tanks, valves, and necessary tubing. The electrical signals were fed into two Brush BL-320 analysers and a Brush BL-202 two-channel oscillograph.

Tests of this assembly mounted on a dump truck provided by the Indiana State Highway Commission indicated that the system was as feasible for heavy vehicles as it was for passenger cars. The same tests also illustrated a new problem created by the much higher pressure used in the truck tires. A blow out of a tire or failure of tubing in the assembly would subject the pressure transducer to a differential pressure far above its capacity. To prevent this occurrence, a relief valve was devised and placed in parallel with the transducer. The valve consisted of a tube with a housing machined to hold an air tight diaphragm which normally blocked the passage. In the event that pressure on one side of the

transducer (and relief valve) should fall 2.5 psi below the pressure on the other side, the diaphragm of the valve would fail, thus equalizing the pressures on sides A and B of the transducer. The valve was so constructed that the diaphragm could be replaced easily and quickly in the field. The final assembly is shown mounted on the test truck in Figure 3.2

Calibration of Force-Pressure Relationship

In order to interpret the tire pressure records in terms of variation of tire force on the pavement, it was necessary to establish a reliable relationship between tire pressure and tire force for the particular wheel and vehicle being used for the test. Both static and dynamic calibration techniques had been used in previous studies (10, 29, 36, 48). In the dynamic methods, both steady state and transient excitations had been tried.

The static approach to calibration has a serious disadvantage in that it gives no information about the response of the system under dynamic conditions. Wilson's work showed a significant dependence of the pressure measuring system for an automobile on the frequency of variation. His theoretical developments indicated that an even greater frequency dependence might be expected in a system for heavy vehicles at high tire pressures. In addition, a system employing a continuous equalization method of temperature compensation could not be calibrated statically because the reference pressure would drift continuously under a sustained load. Hence, it was necessary to rely on some form of dynamic test for calibration of the device.

Both steady state and transient calibration tests were used by Wilson in evaluating techniques for calibrating the pressure measuring system for

FIGURE 3.2. TIRE PRESSURE MEASURING EQUIPMENT MOUNTED ON TEST TRUCK

automobiles. Considerable difficulty was involved in the use of steady state tests in selecting appropriate amplitudes of oscillation. Comparatively large amplitudes were necessary at low frequencies because of the small force variation involved, but the amplitudes had to be decreased as frequency increased in order to prevent losing contact with the surface. Transient tests, though posing some problems, appeared to provide the most satisfactory solution. Both shaped and step pulses were used with only minor differences in results. The step input is achieved by jacking the axle of the vehicle slightly above the force scale and suddenly collapsing the jack. The subsequent dropping of the vehicle sets up a series of oscillations made up of a combination of a wide range of frequencies. This provides optimum response at the higher frequencies in the range of interest but rather low outputs, and some accompanying errors, at the lower frequencies. Somewhat better response in the lower frequency range at a sacrifice in the higher range is achieved by impulsively displacing the force scale on which the tire rests. On the basis of experience with automobiles, it was decided to build a calibrator for heavy trucks employing the step-type impulse for a transient calibration test. This device consisted essentially of a short beam pivoted at one end and equipped with a latch at the other end. An hydraulic jack is used to raise the free end of the beam with the loaded test vehicle in place, and the latch which is adjustable to different heights holds the beam in the raised position. The beam can accommodate single or dual wheels up to a total load of 14,000 lbs. The tires rest on force measuring platforms consisting of specially fabricated plates mounted on load rings. SR-4 strain gages

on the rings provide a continuous measure of force applied to the platform. When the beam latch is released the test vehicle drops suddenly, inducing a transient series of oscillations of force and accompanying variation of tire pressure. Signals from the force platform and the pressure measuring system are fed into a two-channel oscillograph so that simultaneous analog records of force and pressure variation are obtained. Details of the design and construction of the calibrator are described by McLemore (36).

In order to obtain the force pressure relationship in terms of frequency, the force and pressure records are transformed from the time domain to the frequency domain by an harmonic analysis. For each frequency represented, the ratio of force to pressure is calculated. A typical plot of force/pressure ratio versus frequency is shown in Figure 3.3 This figure illustrates the frequency dependence of the pressure measuring system and indicates that it is not feasible to apply an average constant calibration factor to pressure records to obtain force. It is necessary in interpreting field measurements of pressure, then, to transform these records also into the frequency domain, apply the appropriate calibration factor to each frequency represented, and reconvert the resulting record to the time domain. In this way it is assured that each component frequency is multiplied by the correct calibration, and the force record is not distorted by unduly magnifying certain frequencies or attenuating others. The force record is oriented on the highway by event marks on the field record created by the tires moving over an index marker on the pavement. In this manner, the time record is related to vehicle position on the highway so that a record is obtained of force versus displacement of the vehicle along the highway. Figure 3.4 shows a typical record of force for both inner and outer dual wheels of the test truck.

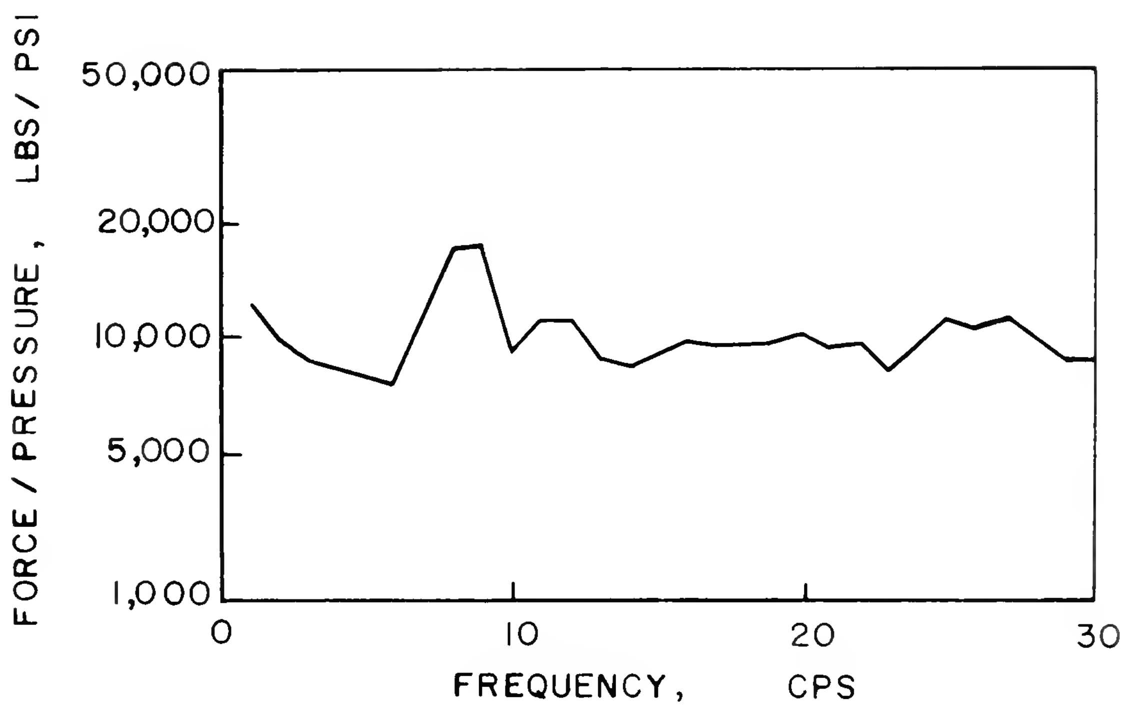


FIGURE 3.3. TYPICAL FORCE / PRESSURE VS FREQUENCY RELATIONSHIP

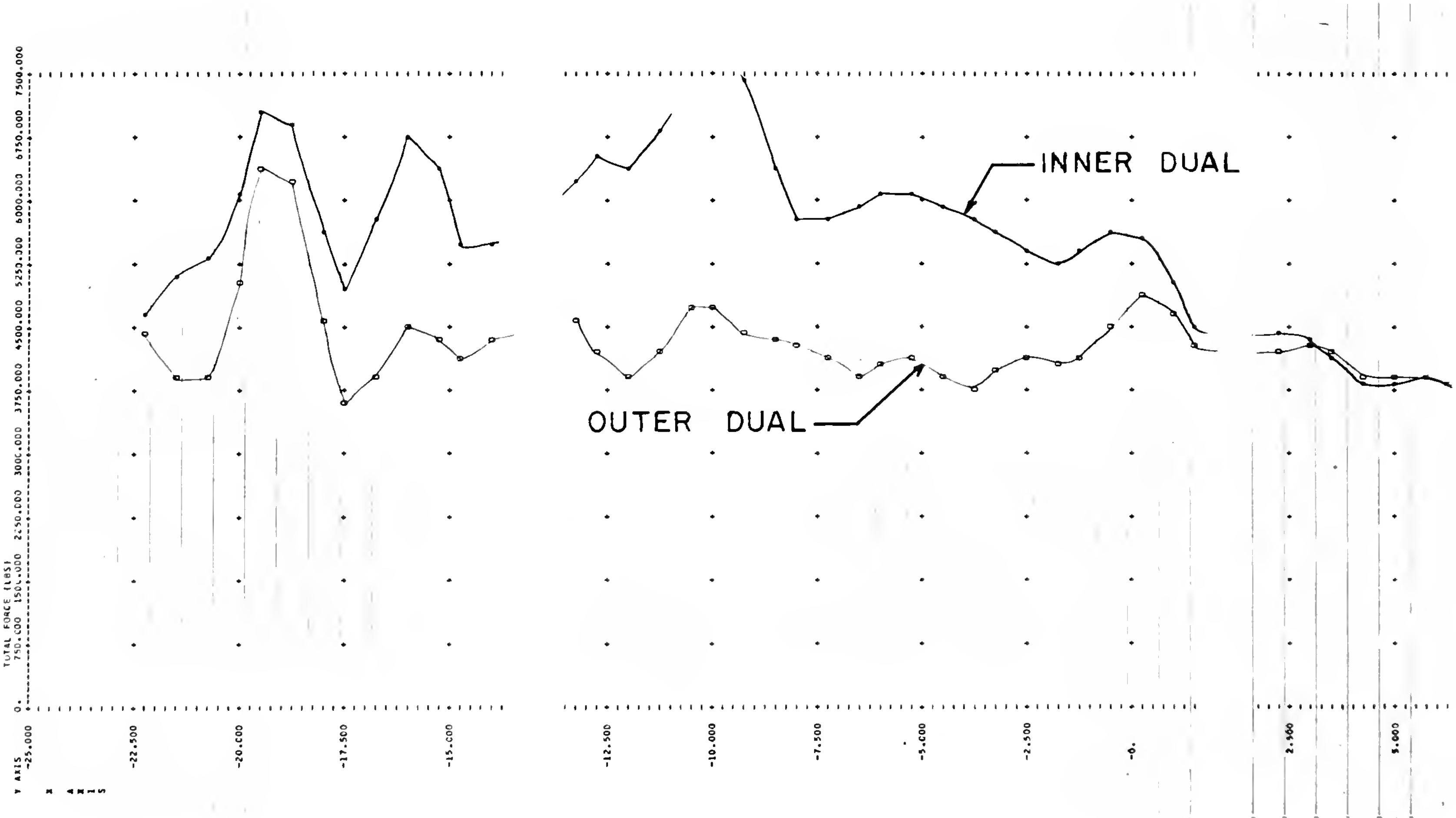


FIGURE 3.4. TYPICAL RECORD OF TIRE FORCE VS LONGITUDINAL POSITION OF TRUCK

4. THE STRESS AND DISPLACEMENT MEASURING SYSTEM

Stress Cells

General Considerations

The measurement of stress in a mass is a difficult experimental problem. In the case of soils, the common techniques of analysing stress from measurement of strain are not generally applicable because of the nature of the material and uncertainties in evaluating elastic properties. It is necessary, therefore, to depend largely on direct measurements of pressure. The fundamental problem with this type of measurement is that the transducer itself introduces a discontinuity in the soil mass; the stress distribution that would exist if there were no discontinuity is altered by the existence of the transducer. In addition, the stress-strain characteristics of the transducer can be expected to differ from those of the mass, and hence the indicated stresses will differ correspondingly from what would exist in the mass in the absence of the transducer. Nevertheless, the need for experimental evidence of pressure distributions in soils subjected to loads has resulted in many attempts to measure stresses directly (9, 16, 23, 26, 27, 28, 30, 37, 42).

The most common approach to this type of measurement is to use a pressure sensitive cell composed of a flexible diaphragm mounted in a rigid supporting frame. The housing is arranged so that one face of the diaphragm is subjected to the surrounding pressure while the other side is

not. Deflections of the diaphragm are sensed by strain or displacement measuring devices and observed on external monitoring equipment. The cell output is then calibrated with the corresponding pressure on the exposed surface of the cell. Benkelman and Lancaster (8) pointed out some of the important considerations in the use of such cells. Later, in an extensive investigation of types and physical characteristics of pressure sensitive cells in soil masses (17) the U. S. Corps of Engineers established specific criteria for design of such transducers, especially with reference to the diameter-thickness and diameter-deflection ratios.

McMahon-Yoder Cells

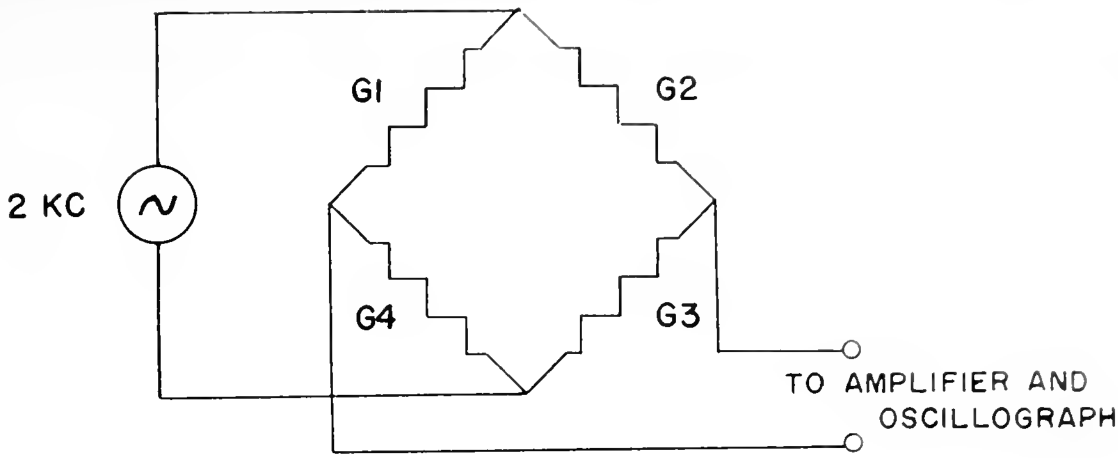
In 1959, McMahon and Yoder (37) reported the development of a pressure cell for use in highway pavements and subgrades. The design followed the criteria established by prior studies while meeting the need for a small, rugged, inexpensive transducer. These cells consisted of a one inch diameter diaphragm and a supporting ring machined integrally of stainless steel. Two SR-4 strain gages mounted on the inside of the diaphragm and connected in series constituted the sensing element. The details of cell construction and measurements of stress distribution in various soil systems under plate bearing loads have been reported in the literature (37).

At an early stage of this project it was decided to use the McMahon-Yoder cells with as little modification as possible. These cells had proved satisfactory in static tests, although the output was too low for satisfactory dynamic response with the equipment then available. A major effort, therefore, was directed towards improving the cells dynamic response characteristics.

The first step in accomplishing the above was to increase the output of the cell itself. It was found that four type A-18 SR-4 gages could be placed on the diaphragm - two at the center and two at the edge. By connecting these gages in a four-arm bridge configuration, as shown in Figure 4.1, the electrical output of the cell was essentially double that of the earlier model. The second step was to assemble suitable electronic equipment for excitation and output amplification so that the output of the cell could be recorded under transient loading conditions with adequate sensitivity for the small stresses that were anticipated in highway applications.

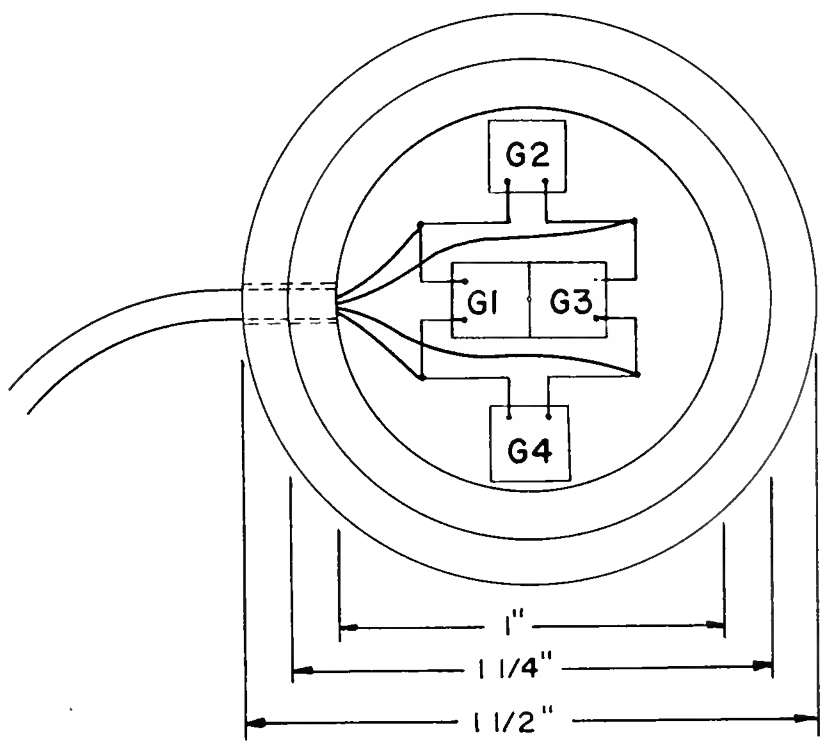
There was available to this project a multiple channel system designed for use with linear variable differential transformers (LVDT). This system consisted of two oscillators, seven amplifiers, a six-channel oscillograph and associated power supplies. The equipment is described in detail in a report of an earlier project by Geldmacher et al. (21). To adapt this equipment for dynamic stress measurements, an isolation circuit was built using an internal 2000 cycles per second oscillator as excitation for the bridge circuit of the stress cells and connecting the cell output circuit to the data amplifiers. The amplifiers were reconditioned and a new filament power supply was built to provide more stable operation. With this equipment, the dynamic response of the cells appeared to be excellent in laboratory tests.

To check this response in a field installation, a typical cell was installed in the subgrade of an unsurfaced pavement. An unloaded station wagon was driven over the cell; the response appeared adequate for the planned tests.



(A)

FOUR ARM BRIDGE GAGE CONFIGURATION



(B)

ARRANGEMENT OF GAGES IN CELL

FIGURE 4.1. SCHEMATIC DIAGRAM OF STRESS CELL AND STRAIN GAGE CONFIGURATION

Calibration

In the original development of the McMahon-Yoder cells, a calibration technique was developed in which the cell was subjected to air pressure through a flexible membrane in a triaxial cell. This basic procedure was adopted for this project with two modifications which were necessitated by the particular application.

One need for modification arose from the use of alternating current (ac) excitation for the strain gage circuit in the cells, and the use of the voltage unbalance as the measure of deflection in the diaphragm (or stress applied). When an ac voltage is used, capacitance in the lead wires introduces a significant impedance in the circuit which is indistinguishable at the output from the resistances of the strain gages. Variations in length of lead can introduce different amounts of capacitance in adjacent legs of the bridge circuit. This variation in capacitance would indicate a voltage unbalance at the output that would appear as a stress reading. To eliminate errors from this source and to simplify field procedures, the calibration was carried out with all leads attached and wired through the same electronic components which would be used in the field. Thus, each calibration was for a complete channel of stress data - cell, leads, and amplifier. A separate calibration cell was built to provide accurate zero setting of the oscillograph during field testing and for setting amplifier gain.

The second, and more significant, problem introduced in the stress cell calibration for this project was the mounting of the cells on top of the transducers for the displacement measuring system. A fundamental purpose of the study was to achieve stress and displacement measurements

at the same point simultaneously. Since completely different transducers were necessary for these two measurements, the manner of installing them was one of the critical decisions which had to be made. Several possibilities were considered. The simplest method considered, from the point of view of installation and testing, was to mount the two transducers together as a single unit. As pointed out by McMahon and Yoder (37), however, if the diameter-thickness ratio for the stress cell were not kept large - near five or greater - large discrepancies would exist between actual and indicated stresses. When mounted together, the stress cell and accelerometer constituted a single unit of diameter-thickness ratio of approximately 2/3. This would result in indicated stresses considerably larger than true values. To avoid this difficulty, consideration was given to placing the transducers in separate locations along a longitudinal or transverse line in the pavement. Positions on a longitudinal line would theoretically be subjected to the same load at different times. Positions along a transverse line would be subjected to different loads on individual runs, but by a judicious choice of lateral positioning of the test load, geometrically similar loading could be produced for both transducers in different runs. Neither of these alternates was attractive because of the uncertainties of homogeneity of the pavement and subgrade, as well as variations of load, velocity, and lateral position of the test vehicle in successive runs. It was believed that errors introduced by these variables would probably be greater than those produced by the undesirable proportions of the combined instruments. In addition, of course, the problems of installation would be only half as great for a single unit as for two separate transducers. It was decided, therefore, to use the stress cell mounted on the accelerometer as shown in Figure 4.2.

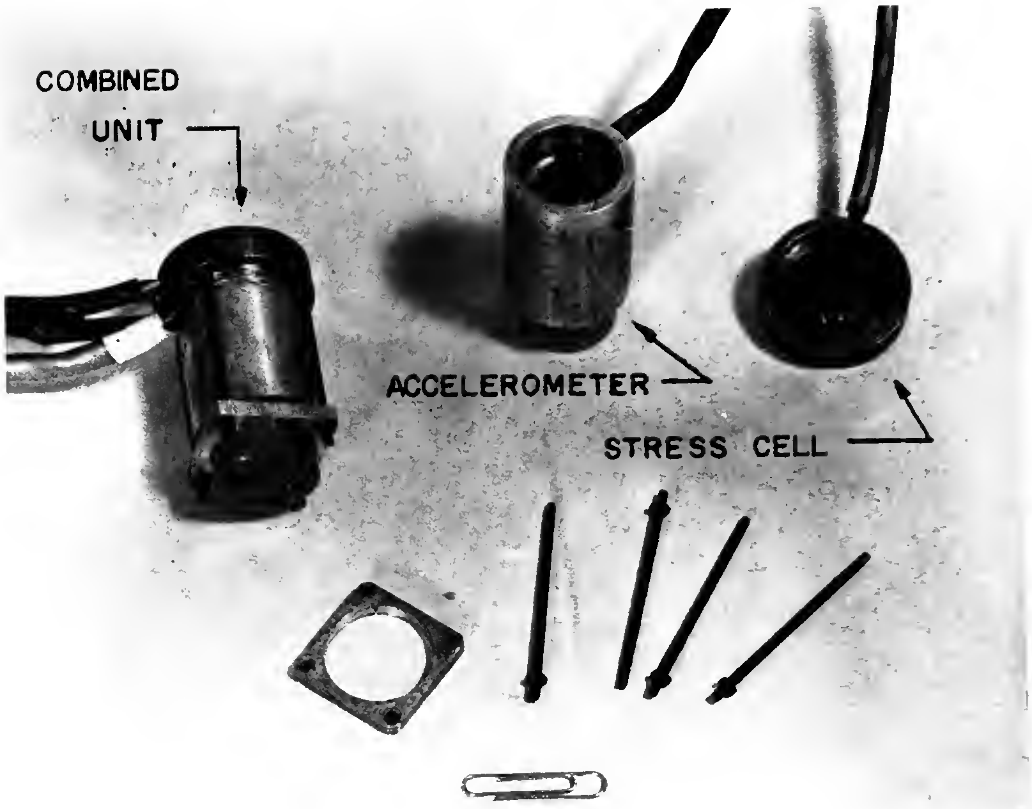


FIGURE 4.2. STRESS CELL AND ACCELEROMETER

In order to correct for the low diameter-thickness ratio, a number of tests were run on the cells in the mounted and unmounted conditions. After laboratory calibration in the triaxial cell, one stress cell was mounted on an accelerometer and another prepared for embedment as a separate unit. Two installations were made in an unsurfaced section of road, one at a depth of six inches (at the interface between soil subgrade and gravel base) and one at a depth of 22 inches (in the subgrade). For each installation, both transducers were embedded, three feet apart, in one wheel path. Observations of stress were then made using an unloaded test truck. Records were made of stress under static load with the load placed first over one cell, then over the other cell. Runs were also made at creep speed and about 10 miles per hour in which the truck moved over both cells. Higher speeds were not possible at that test site. Stresses were so low at the 22 inch depth that only two trials yielded readable deflections of the oscillograph record. Some of the runs on the six inch depth test were invalidated by irregularities in the road surface which caused the load to shift from one of the dual tires to another in passing from the location of one transducer to the next. However, the stresses indicated by each of the cells for each valid load were obtained by referring to the calibration curves obtained in the laboratory. A plot of stress indicated by the unmounted cell versus stress indicated by the cell mounted on the accelerometer is shown in Figure 4.3.

Considerable variation was expected in the aforementioned points, at least for the six inch depth, because experience with the cells in the original work indicated a somewhat erratic response in granular soils. Even with this variation it was considered that the indication of stress

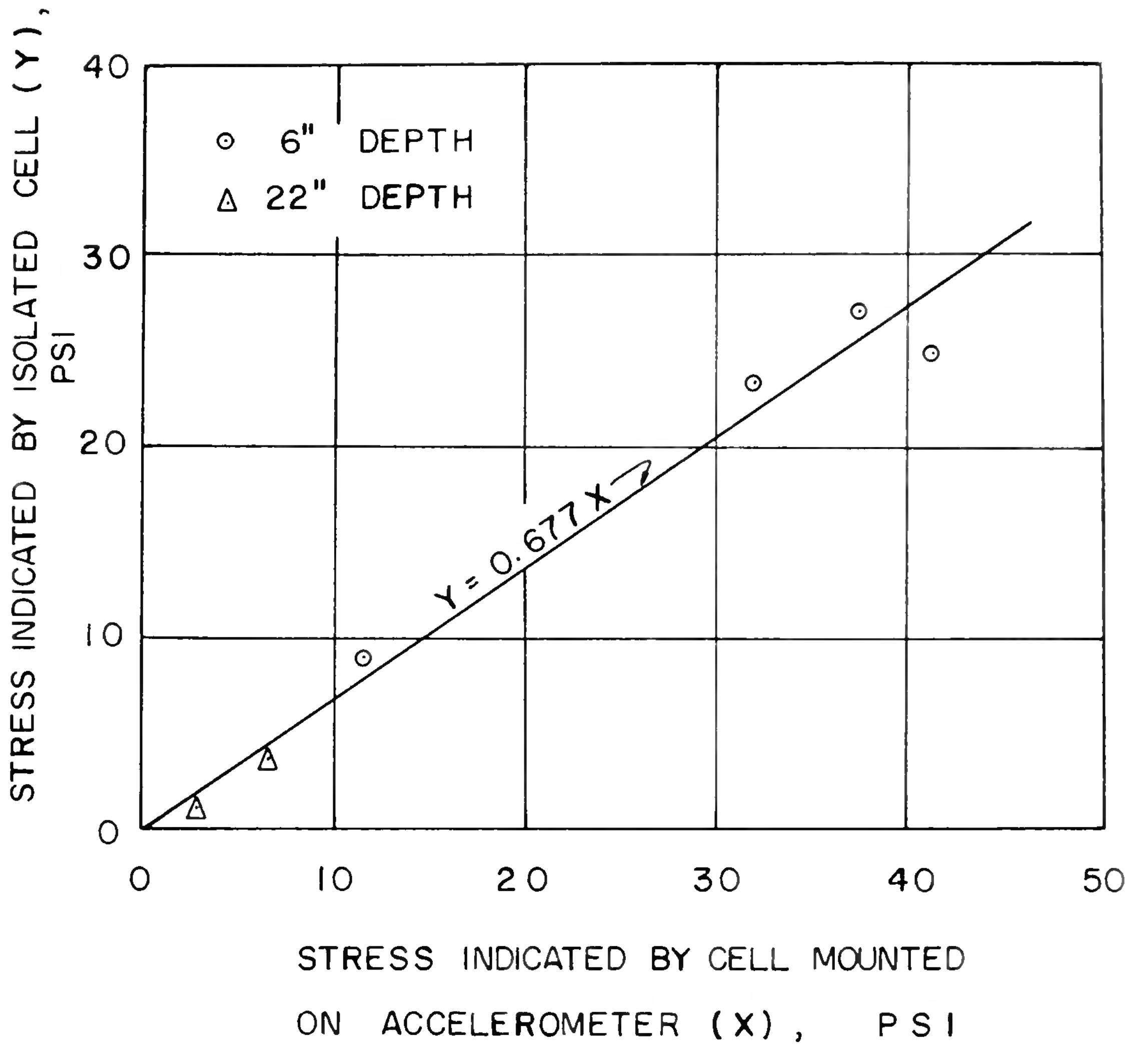


FIGURE 4.3. STRESS INDICATED BY CELLS UNMOUNTED AND MOUNTED ON ACCELEROMETER

patterns was valid, and from these data it appears reasonable to assume a linear relation between the two methods of installing the cells. The line shown in Figure 4.3 was therefore determined by fitting to the points a straight line through the origin using the method of least squares. The interpretation given this relationship is that the actual stress under field loading conditions can be taken as 0.677 times the stress indicated by a pressure cell which is mounted on an accelerometer used in this study.

With this procedure developed, the stress measuring system was considered usable. All field tests made in the summer of 1964 used stress cells and accelerometers mounted as a single unit. Since the accelerometer was in this way forced to move identically with the stress cell, it is assumed that the measured deflections (as described in the next section) occurred at precisely the same points at which the stress was measured.

Displacement Measurement

General Considerations

Methods of Measuring Displacements. Many attempts have been made to measure displacements of pavements under highway type loads. Among the first was Hveem's (32) use of an electric travel gage embedded in the pavement surface and referenced to a rod driven into the subgrade. This basic technique has been used by others with various refinements, notably the use of LVDT's, as the displacement transducer (26, 27, 28, 30). At the WASHO Road Test, Benkelman developed a rotating beam device which was simple and inexpensive for measuring displacements at the pavement surface.

Details of the device are described by Carey (14). Walker et al. (44) applied this device to the measurement of displacements of pavement components and subgrades. A number of other studies also have applied the same device, in some cases using modified techniques which were claimed to improve its reliability (13, 30, 31, 47, 49). Baker (4) describes the application of photogrammetric techniques to the problem of transient pavement displacements.

All of the pavement displacement studies discussed in the literature rely on some form of one of the three techniques mentioned above. Yet each of these methods involves one or more of a number of severe disadvantages. The photogrammetric technique is limited to displacements of the pavement surface. The Benkelman Beam cannot measure displacements directly under the load since the device must be operated on the pavement, restricting not only the position of the load with reference to the point of measurement, but also the vehicle velocity at which tests can be run. Displacements under static loads or loads moving at creep speed can be made conveniently with the Benkelman Beam, but tests at high vehicle speeds are impossible. These difficulties can be overcome by installing a displacement transducer in the pavement but several new problems are introduced. The first of these is the existence of a hole in the permanent structure introduced to house the displacement transducer and isolate the pavement from the embedded datum rod. It is not known how much effect this hole has on the behavior of the pavement under load. In fact, it seems reasonable to expect that this effect would be highly variable, depending on the size and depth of the hole and the type of casing. The manner of establishing a reference (that is, embedding a plate or driving

a rod) and the method of transferring pavement movements to the movable part of the transducer in the hole might also affect the indicated displacements. A more serious shortcoming even, as far as this project was concerned, was the inability of measuring stress at the same point as displacement if there is a hole permanently established at that location. It is evident that the stress distribution in the vicinity of such an installation would be significantly different from that in an undisturbed pavement even if the displacements were not greatly affected. Therefore, stress and displacement measurements would have to be made at widely separated points. In addition, the establishment of a fixed reference is a questionable process at best. Geldmacher et al. (21) showed that in some cases reference rods driven as much as 40 feet deep beneath a rigid pavement showed significant movement (seven to eight milli-inches) under highway type loadings. With the greater stresses generally observed in subgrades beneath flexible type pavements it is reasonable to expect even greater movements, indicating considerable uncertainty as to actual effectiveness of any practical datum that might be installed.

For these reasons, it seemed worthwhile investigating an inertial type system for sensing pavement displacements. Such a system would require no permanent hole in the structure, no fixed datum to be established within the pavement, could be installed at any depth in the structure or subgrade and could be used for any vehicle location and (theoretically, at least) any velocity except zero. Ideally, such a transducer would be small, rugged and inexpensive. The first two of these criteria could be met reasonably well. The cost of instruments meeting these requirements and also having suitable frequency characteristics and resolution is less favorable.

The Inertial System. The basis for any inertial type of transducer is the spring suspended mass, usually with viscous damping. This system is illustrated in Figure 4.4 in which the mass W is suspended from a frame or case, a, by a spring of constant k (the ratio of load to deflection of the spring). The dashpot, c, exerts a force on the mass proportional to the relative velocity of the mass and case. The scales x and y indicate the displacements of the case, and the mass relative to the case, respectively. The theory of operation of such instruments is covered in detail in many standard texts on vibrations and experimental measurements, as for example, Timoshenko (43) or Dove and Adams (18).

The differential equation of motion of the suspended mass, W, in Figure 4.4 is

$$\frac{d^2y}{dt^2} + \frac{cg}{W}\frac{dy}{dt} + \frac{kg}{W}y = -\frac{d^2x}{dt^2} \qquad 4.1$$

in which

y = displacement of the mass relative to the case

t = time

g = acceleration of gravity = 32.2 ft/sec^2

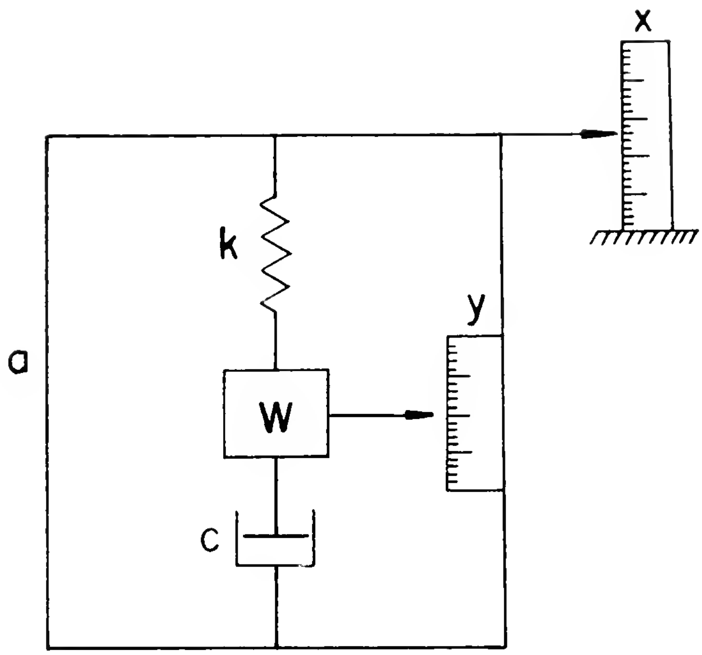
c = dashpot constant = ratio of damping force to velocity

k = spring constant = ratio of load to extension

W = weight of mass

x = displacement of case.

Timoshenko gives the solution of this equation (in slightly different form) for the case where the case is given a periodic motion as



MASS-SPRING-DASHPOT SYSTEM

FIGURE 4.4. SCHEMATIC DIAGRAM OF MASS-SPRING-DASHPOT SYSTEM

$$y = e^{-nt}\left(A \cos \sqrt{p^2 - n^2}\; t + B \sin \sqrt{p^2 - n^2}\; t\right) + \frac{\left(\frac{\omega}{p}\right)^2 X \sin(\omega t - \alpha)}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \frac{4n^2\omega^2}{p^4}}} \qquad 4.2$$

in which

A and B are constants determined from initial conditions,

$p^2 = \frac{kg}{W}$

$2n = \frac{cg}{W}$

$\alpha = \tan^{-1} \frac{2n\omega}{p^2 - \omega^2}$

ω = circular frequency of the motion x, and

X = the magnitude of the forcing motion

While a consideration of response to a periodic motion is not adequate for evaluating response to transient displacements, some important generalizations can be observed from an examination of equation 4.2. The displacement y of the mass relative to the case is composed of two parts - the first term representing the initial disturbance to the system, and decaying more or less rapidly, depending on the value of n which represents the damping force, and the second term representing the steady state conditions. It is seen that the latter term is greatly dependent on the ratios ω/p and n/p. The significance of p can be seen by considering an undamped system, corresponding to the case where $n = c = 0$, subject to free vibration. Under these conditions, the last term, which arises from an external disturbing force, disappears, and the equation 4.2 reduces to

$$y = A \cos pt + B \sin pt \tag{4.3}$$

This indicates a periodic motion recurring at a circular frequency of p radians per second. Thus, p represents the natural, or resonant, frequency of the undamped system. As noted above, ω is the circular frequency, in radians per second, of the case motion.

Returning to the last term of equation 4.2, the effect of the ratio ω/p of the frequency of the forcing motion to the resonant frequency can be inferred. In practical instruments the ratio n/p is very small and can be considered to be zero (43). If $\omega/p = 1$ then, the factor

$$Y_1 = \frac{(\frac{\omega}{p})^2}{\sqrt{\left[1 - (\frac{\omega}{p})^2\right]^2 + \frac{4n^2\omega^2}{p^4}}} \quad \text{approaches } \infty, \tag{4.4}$$

indicating that relative deflections of the mass will be extremely large in practice, much larger than the deflections of the case. In this event the range of motion of the mass is quickly exceeded and the device is not useful for indicating the magnitude of the forcing motion. If ω/p gets very large, Y_1 approaches one, which indicates that the steady state displacement, y, of the mass, as initial effects in the first term are damped out, approaches the forcing displacement, x. This arrangement is a displacement transducer, and is therefore a desirable device for measuring deflections of certain frequencies. However, in order to have ω/p suitably large when measuring displacements of only one or two cycles per second frequency, the resonant frequency of the instrument must be extremely small. For example, if ω/p is only 2 and $\omega = 1$ radian per second, p must be 0.5 radians per second. Taking a mass weighing 0.1 lb, this would require

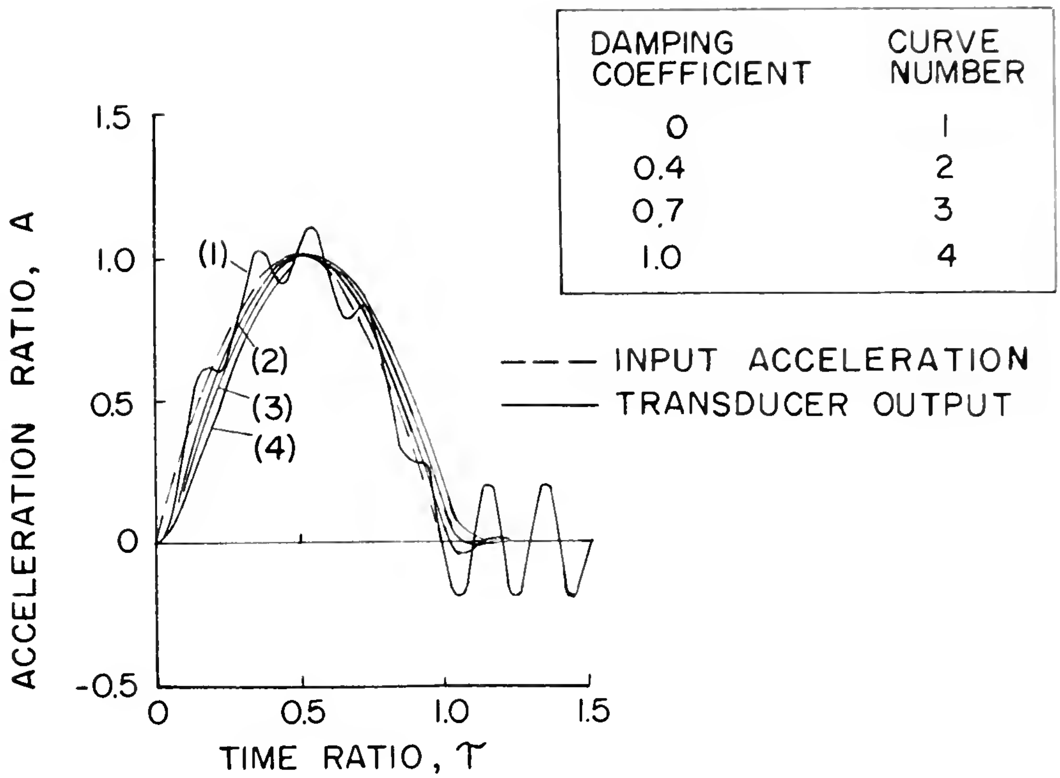
a spring which would extend 129 feet merely to support the mass under the influence of gravity. It clearly is not feasible to provide a small transducer with such low natural frequency as is required for this condition.

Considering, on the other hand, a ratio ω/p much less than 1, the factor Y_1 approaches zero. Under these conditions, once the transients are damped out the suspended mass goes through precisely the same motion as the case, and there is no relative motion at all. The force acting on the mass to cause it to follow the case motion is transmitted through the spring suspension, and from Newton's principle it can be seen that this force is proportional to the acceleration of the mass (which is the same as that of the case, since the motion is identical). Thus, if a device is so designed that the force in the spring can be monitored constantly, and the natural frequency can be kept several times as great as the frequency to be measured, a signal proportional to acceleration of the motion to be measured can be obtained. Since natural frequency is proportional to spring stiffness and inversely proportional to weight of the suspended mass, construction of an acceleration transducer, or accelerometer, is not so formidable a task as construction of a displacement transducer for measuring motions in the frequency range from about one to fifteen cycles per second.

Since acceleration is the second derivative of displacement with respect to time, it should be possible to obtain a record of displacement of a forcing motion by double integrating the output signal from an acceleration transducer. Furthermore, if an accelerometer could be obtained which would reproduce transient inputs with reasonable fidelity, it might be useful as a transducer for measuring pavement deflections.

The response to transient conditions, however, is much more varied than that to steady state conditions. As in any application, the response is governed by the equation of motion for the system, equation 4.1, but in the case of transients it is not adequate to consider only the steady state response. It is necessary to obtain the entire solution for each input motion to be investigated. Many studies of this (18) have shown that the accuracy of the response is greatly affected by the rise time of the forcing motion as well as by the characteristics of the accelerometer. In general it can be stated that an accelerometer with appropriate damping and natural frequency will reproduce best those accelerations having the most gradual rate of increase. Figure 4.5 illustrates the response to a half sine pulse of acceleration obtained by Levy and Kroll (35) for various damping ratios of an accelerometer whose natural frequency is five times as great as the frequency of the input.

In the absence of any previous field data on the actual accelerations involved in pavement deflections, it appeared reasonable to expect even less demanding an input acceleration than that used by Levy and Kroll. Figure 4.6 shows a comparison of a deflection profile plotted from AASHO Road Test data (30) and a cosine curve drawn to the same length and maximum deflection. It is evident that the actual deflection rises more slowly near the extremities, growing more rapidly near the center, than the cosine curve. This would be expected of a deflection occurring from an acceleration which rises gradually from a zero slope at zero acceleration, such as a moving vehicle would produce as it approached a given point. Figure 4.7 contrasts the instantaneous increase in acceleration required to produce a cosine deflection with the gradual acceleration which would produce the actual deflection pattern shown in Figure 4.6.



T = DURATION OF INPUT PULSE

$$\tau = \frac{t}{T}$$

$$A = \frac{d^2y}{dt^2} \Big/ \left(\frac{d^2y}{dt^2}\right)_{max}$$

FIGURE 4.5. ACCELEROMETER RESPONSE TO TRANSIENT PULSES (FROM LEVY AND KROLL, <u>JOURNAL OF RESEARCH</u>, BUREAU OF STANDARDS, 1950)

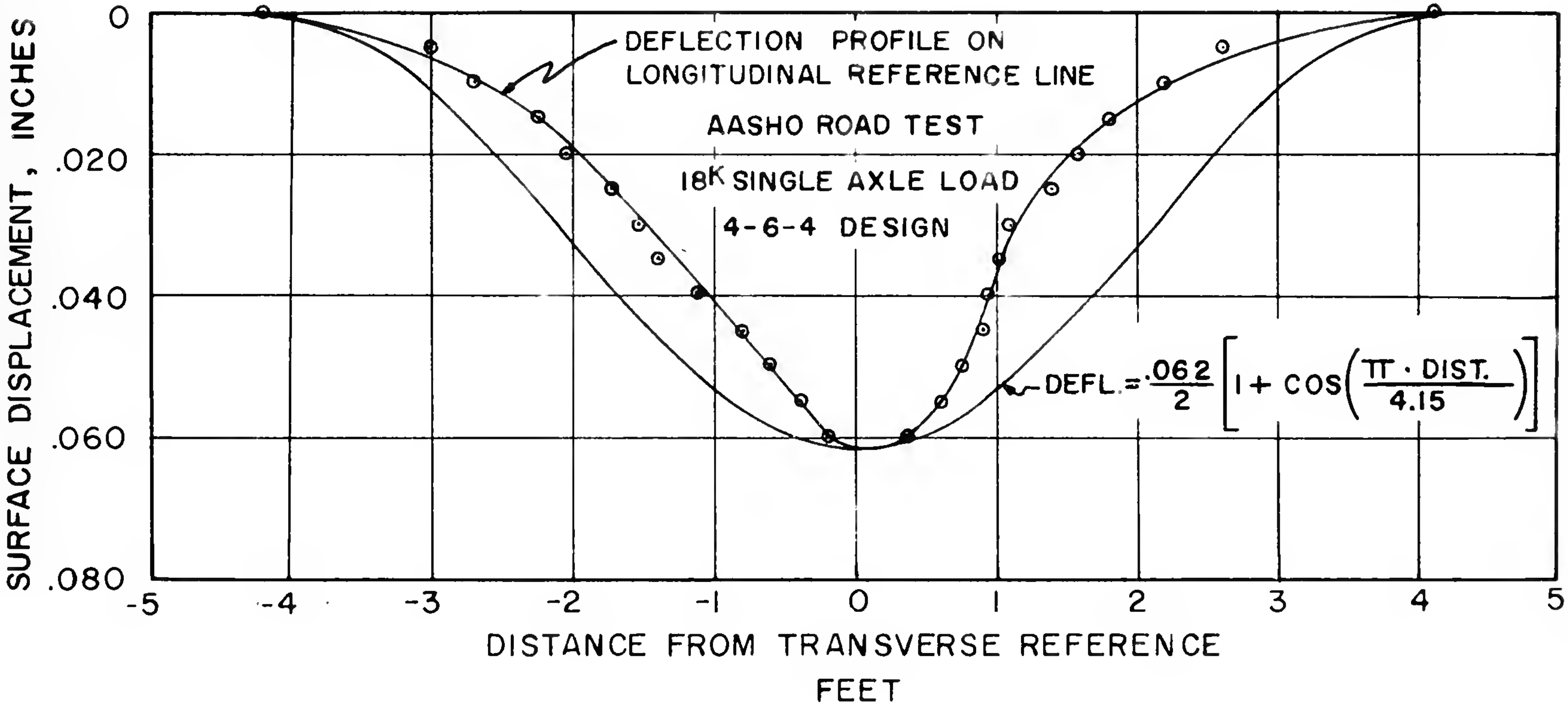


FIGURE 4.6. COMPARISON OF REAL AND THEORETICAL DISPLACEMENTS

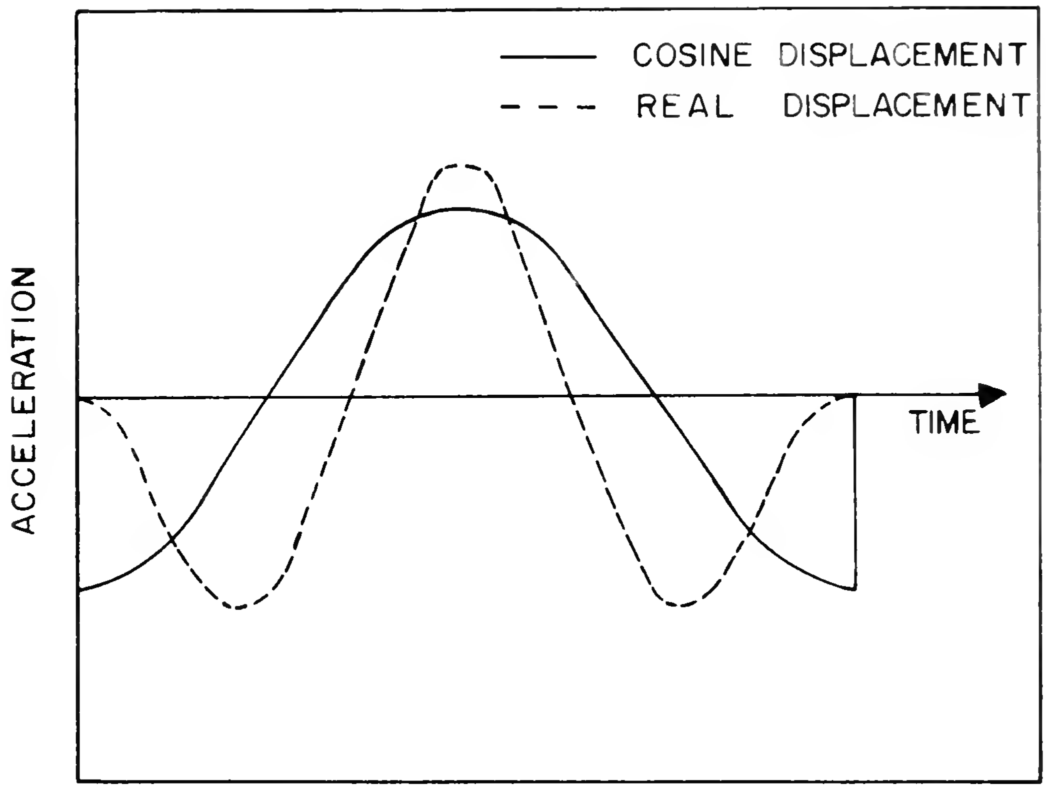


FIGURE 4.7. COMPARISON OF ACCELERATIONS ASSOCIATED WITH REAL AND THEORETICAL DISPLACEMENTS

Actual pavement deflections, then, could presumably be reproduced even better by the accelerometer than could the sine pulse in Figure 4.5. The technique of obtaining acceleration measurements and double integrating the signals to obtain displacements introduces difficulties, especially in as much as the deflections to be measured are extremely small and of comparatively low frequency. The advantages of the inertial system, however, appeared to be great, and electronic components had developed to the point that this approach gave some promise of feasibility. The success of the Association of American Railroads in applying this system to the measurement of railroad car displacements (2) lent strong support to the proposal.

Laboratory Development

The attempt to adopt an inertial type of transducer to the measurement of pavement deflections involved three major problems: 1) a convenient means of testing and evaluating the equipment in the laboratory, 2) locating a transducer with appropriate frequency characteristics and suitably low resolution and 3) developing computation and recording equipment. Laboratory testing would be important to permit quick and easy trials of different conditions, as well as providing suitable excitation for work on the electronic systems required for amplification, integration and recording. It appeared more promising to find a commercially produced transducer that would meet the conditions of this project than to try to build one for the specific application. Efforts in developing the electronic components were directed at making as much use as possible of equipment already available.

Laboratory Test Table. For purposes of testing in the laboratory, a simple means of holding a transducer and moving it through a small vertical displacement in an interval of say one to one-fifteenth of a second was needed. There was available a small table which had been used for making flow tests of hydraulic cement. The apparatus as originally built, except for the manual drive, is described in detail in American Society for Testing and Materials Standards (1). The device consisted of a ten inch diameter table on a vertical spindle mounted in a frame. The bottom of the spindle rested on a cam which, when rotated, raised the table 3/4 of an inch, then dropped it suddenly to its lowest position. This cam was replaced for purposes of this project by an eccentric circular cam which, as it was turned, would raise and lower the table smoothly through a total vertical displacement equal to twice the eccentricity of the cam.

For original trials, a cam one inch in diameter with a 0.1 inch eccentricity was used. This produced a total displacement of 0.2 inch in a full revolution which is considerably larger than displacements anticipated in the field. However, smaller deflections could be obtained by using partial revolutions of the cam, and accelerations could be kept low by operating the table at low speeds. This combination provided flexibility enough so that transducers could be tried over a considerably wider range of conditions than would be expected in practice. After selection of a transducer, a cam with eccentricity of about 0.05 inch was used.

In early trials of transducers, it became apparent that the vertical shaft did not run smoothly on the rotating cam. A high frequency chatter was introduced into the vertical motion which masked the gross motion which was to be measured. To overcome this, a new shaft was made with a

roller bearing in the end which rode on the cam. A spline was also provided in the new shaft and a matching key in the frame, to prevent rotation of the table in the horizontal plane.

This apparatus provod satisfactory for all testing and calibration. The only shortcoming was that with manual operation it was not possible to get a full revolution in an interval less than about 1/3 to 1/4 of a second. Higher frequency testing was possible, however, with partial revolutions of the cam. Transducers were placed directly on the table or mounted on special holding blocks, according to the physical arrangement of the unit. Power and signal leads were fastened so as not to interfere with the motion or to disturb the transducer during vertical displacements.

It was necessary to establish a time record of table displacement simultaneously with the transducer output for purposes of comparison. This was accomplished by mounting an LVDT as a displacement transducer and recording the output on a separate channel from that of the inertial transducer on the same oscillograph. Calibration of the LVDT was accomplished by comparing oscillograph pen deflections with table displacement measured with a micrometer dial. The complete apparatus, including the modified flow table, LVDT, dial gage and an accelerometer in position for test, is illustrated in Figure 4.8.

Transducers. The next problem was to locate a transducer with suitable frequency characteristics and adequately low resolution. Investigation of existing pavement deflection data and consideration of proposed vehicle operating speeds had given an indication of characteristics of the motion that might be expected. Deflections would probably range from a few

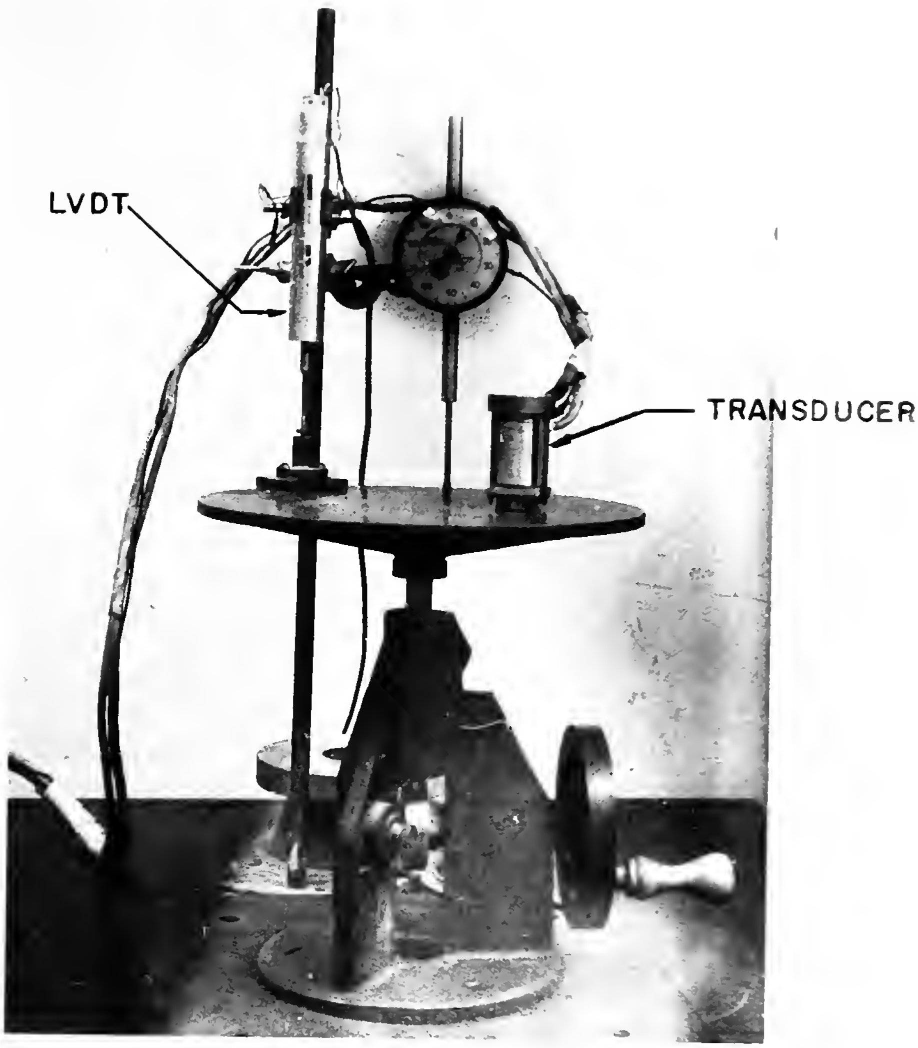


FIGURE 4.8. LABORATORY TEST TABLE WITH ACCELEROMETER AND CALIBRATING EQUIPMENT

thousandths of an inch to several hundredths. Depending on vehicle operating speed and the range of influence of a wheel load, the deflection and rebound would occur in an interval of one second (or greater for loads approaching static conditions) to one fifteenth second. In other words, the motion would have a frequency in the order of one to fifteen cycles per second. Assuming a sinusoidal deflection, peak accelerations would vary from about 0.0001 g for 0.002 inch displacement at one cycle per second to 0.6 g for 0.05 inch displacement at fifteen cycles per second.

As noted previously it is probably not possible to build an inertial transducer with a natural frequency low enough to be feasible for direct displacement measurement of such motion as was expected in pavements. Nevertheless, an attempt was made to apply a geophone to this service since it was available and it was possible that it would give some further indication of how inertial transducers would respond to the proposed use. The specific unit available had a nominal natural frequency of 18 cps. It was designed only to indicate the time of arrival of vibrations in the earth at the location of the geophone in seismic methods of soil exploration. For this reason, it is sensitive to extremely small motions. Like any inertial transducer it consists of a mass suspended on a spring and subjected to a viscous damping force. Unlike the displacement and acceleration sensitive devices above, however, its output depends on the relative velocity of the mass and the case. Thus, if the frequency ratio ω/p is large, as with the displacement device, the movement of the mass is negligible. The relative velocity of the case and the mass is then the actual velocity of the case, or of the forcing motion. Such a transducer would be preferable to an accelerometer inasmuch as its output would have to be integrated only once.

Trials were made with the borrowed geophone on the laboratory test table previously described. Tests were run at a variety of displacement magnitudes and frequencies. It was evident, however, that for forcing deflections of such low frequency as anticipated in pavements no usable response was possible. Further study of the instrument indicated that its actual natural frequency was close to 16 cycles per second. The wildly erratic output which had been obtained indicated that the natural frequency of the geophone was being excited and confirmed that a useful transducer would have to have a frequency much farther removed from that of the motion being measured.

The advantages of a velocity sensitive instrument over an accelerometer prompted continued search of the literature and commercial products. The smallest velocity meter of suitable frequency and resolution which could be found was about three inches in diameter and three feet long. This approach was then discarded and the search continued for a suitable accelerometer.

Commercially available accelerometers which showed promise of being useful were of three types: piezoelectric, strain gage, and servo-accelerometers. Of these, only the last proved at all satisfactory. The piezoelectric accelerometers depend for their output on a crystal which produces an electric potential when strained. The seismic mass in such an instrument is supported by a piezoelectric crystal. Under the influence of an acceleration the mass exerts a force proportional to the acceleration on the crystal, straining it and producing a charge. This charge, however, is only momentary, and is produced only with a change in the strain condition of the crystal. Such an accelerometer gives no indication of

sustained accelerations (dc conditions) and generally has a rather high frequency as the lower limit of its useful range. In addition, the output is extremely small - a few tens of millivolts per g input. Tests were made using a piezoelectric accelerometer having a usable frequency range of 3 to 12,000 cps and a resolution to 0.05 gs. It was not possible, however, to get any usable output from this transducer on the laboratory test table. Manufacturers of these instruments were consulted, but none would recommend this type of transducer for the proposed use.

The strain gage accelerometers were also investigated. In these instruments wire strain gages are used to sense the force exerted between the mass and the case. For frequencies well below resonant frequency, the output of the strain gage circuit is proportional to the force of acceleration. These instruments have the advantage of a frequency response right down to dc, and in this respect are ideally suited to the proposed application. However, the output is comparatively low, as in the case of piezoelectric units, and the resonant frequency is generally too low.

The third type - the servo-accelerometer - proved to overcome the objections of the other types. Two manufacturers recommended units of this type, and one provided an instrument on a loan basis for evaluation in the laboratory. This was a Kistler Model 304 adjusted for a full scale range of $\pm$ 1g. The specified natural frequency was 150 cps and the resolution was 0.0001 g. The output signal was 5 volts per g. The servo-accelerometer operates on the servo principle of using an "error" signal to restore initial, or "equilibrium", conditions. The seismic mass is suspended in a magnetic field and is attached to a displacement sensor.

A displacement of the case produces a relative displacement of case and suspended mass which causes the sensor to generate a signal. This signal energizes a force coil which exerts a force on the mass just sufficient to restore it to its equilibrium position, thus, producing the same motion in the mass as in the case. The current required to produce the restoring force is proportional to the acceleration of the motion. A resistance across this circuit converts the current to a voltage signal. Varying the value of this load resistor varies the range and sensitivity of the instrument.

Trials of the Model 304 on the test table gave excellent response over a wide range of displacements and frequencies. With high quality amplification, motion even slower than one cycle per second was easily observed. No attempt was made to measure independently of the test transducer what the actual input accelerations were. However, integrated output signals were obtained which agreed qualitatively with the input displacement as recorded by the LVDT. Problems were experienced with the electronic components for performing the integration, but the transducer response appeared to be excellent. Some trials were then run using only one stage of electronic integration, resulting in an oscillograph record of velocity of displacement. This record was integrated manually with a planimeter and the resulting displacement compared to the LVDT record. Some variability existed, but the indications were that the transducer response, at least, was satisfactory and the system appeared feasible. The Kistler Model 304 Servo Accelerometer was, therefore, adopted for the project and efforts were directed toward further development of the electronic amplification and integration.

Analog Computer. Three basic approaches were theoretically available for operating on the acceleration signal which was obtained from the transducer. The signal could be 1) recorded directly and integrated manually, 2) digitalized electronically and integrated on an electronic digital computer, or 3) integrated continuously in an analog computer. Manual integration was originally considered not feasible because of the extensive man hours which would be required and the inaccuracies involved in plotting the first integral of acceleration to produce a curve of velocity which would itself have to be integrated. Electronic digitalizing equipment would be extremely expensive, and like manual integration, would produce no deflection data directly in the field. Final results would be delayed until processing could be done at a computing laboratory. In contrast to these possibilities, the analog computer approach would give an actual displacement record directly in the field, would not involve manual integration and would not require expensive equipment.

The utility of the electronic analog computer is based on the characteristics of certain networks involving the use of high gain direct current (dc) amplifiers. These instruments are capable of providing amplification of magnitudes from 10^4 to 10^9 or higher, and for a given amplifier, the gain is independent of frequency from dc to several thousand cycles per second or greater. The theory of computing networks is explained in many standard works on electronic analog computation as for example, references 34, 38, 41, 45.

A block diagram of the basic network employed is shown in Figure 4.9 in which generalized impedances Z_i and Z_f are indicated. The amplifier

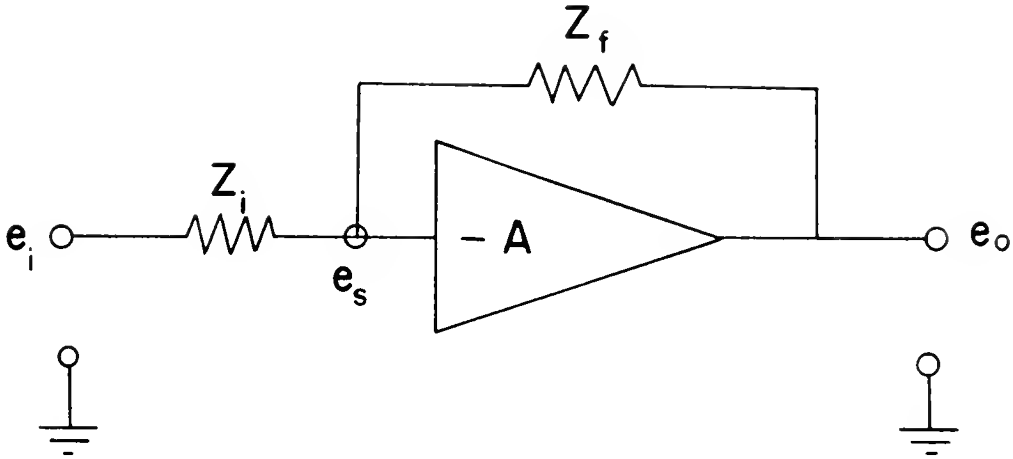


FIGURE 4.9. SCHEMATIC DIAGRAM OF ELECTRONIC ANALOG COMPUTING NETWORK

gain is -A, and the input impedance is so high that it can be assumed that no current flows into the amplifier. The relationship between output and input voltages is then given by

$$e_o = \frac{-\frac{Z_f}{Z_i} e_i}{1 + \frac{1}{A}\left(1 + \frac{Z_f}{Z_i}\right)} \qquad 4.5$$

When A is suitably large, this approaches the approximate relation

$$e_o = -\frac{Z_f}{Z_i} e_i \qquad 4.6$$

If Z_f and Z_i are both pure resistive impedances, the relation becomes

$$e_o = -\frac{R_f}{R_i} e_i \qquad 4.7$$

so that the output voltage is proportional to the input voltage in the ratio of feedback to input resistance. In this way the network is used simply as a dc amplifier to provide any desired gain. On the other hand, if the feedback impedance is a pure capacitance, the relation becomes

$$e_o = -\frac{1}{CR} \int e_i dt \qquad 4.8$$

Thus, the output voltage of the network is proportional to the integral of the input voltage with respect to time. If e_i is a voltage proportional to pavement accelerations, for example, e_o would be proportional to the velocity of the pavement motion. Two of these networks in series will of course provide a double integration of the input.

Tests of such a computing network were made on an educational model analog computer using a signal generator to provide the input voltage. The terminology used in discussing the tests is illustrated in Figure 4.10. Sketch (a) shows how a varying voltage is expressed as the sum of two components, a dc level representing the average voltage and an ac component representing the variation above and below the dc. Sketch (b) shows a transient pulse along with corresponding actual and theoretical responses of a single integrator to this input.

Excellent shapes of output voltage waves were obtained for sine wave and square wave input signals both for single and double integrations. The dc level of the output was not steady, however, as a result of the tendency of the amplifiers to drift. This drift is the error between the theoretical and actual outputs in Figure 4.10b and is the major obstacle to direct coupled electronic integration. It occurs because there is no way for the integrating network to distinguish between signal voltages at the input and any error voltages that arise from spurious noise and current drawn by the amplifier. The usual way to eliminate errors resulting from drift is to ac couple the computing networks, that is, to place a capacitor in series between the signal voltage and the computing network. Since the capacitor will transmit only varying voltages, any dc error in the signal is blocked and only the ac component of the signal is computed. The voltage transmitted by a capacitor, however, is attenuated by the impedance of the capacitor, and is a function of frequency. High frequency signals are passed at almost no loss in voltage, but lower frequency signals are greatly reduced. This effect results in a distorted signal, containing an error

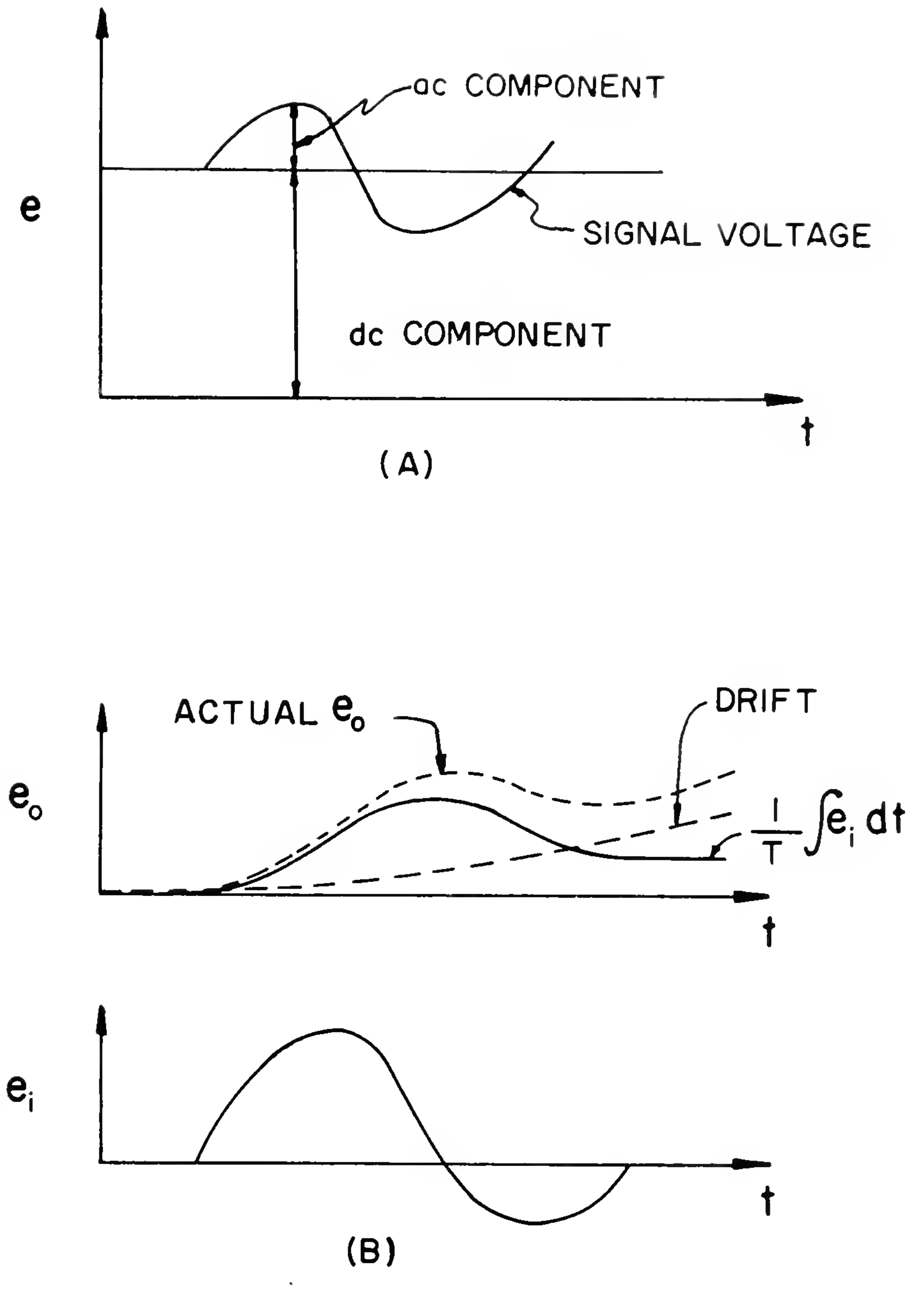


FIGURE 4.10. TERMINOLOGY AND CHARACTERISTICS OF ELECTRIC SIGNALS

the magnitude of which depends on the frequency. For very slow variations, the errors may completely mask the correct results.

An attempt was made to use ac coupling on the analog computer. At frequencies of a few cycles per second, however, the results were so distorted as to be impossible to calibrate.

Since it was necessary to use direct coupled components, filters were tried in the feedback circuit of the integrators. These filters fed back the slowly varying drift voltage to the input to counteract the error which caused it. This improved the stability of the first stage of integration, but had little effect on the second stage. Further improvement was achieved by providing a separate and very stable power supply with a vernier control to balance the dc component of the signal. A schematic diagram of this arrangement is shown in Figure 4.11.

In the configuration of Figure 4.11, the signal is first amplified, then integrated twice by two integrators in series, and finally amplified again, to a level high enough to drive the oscillograph. Tests were run with this setup on the computer using the Kistler accelerometer activated by the test table. In its normal position the accelerometer produced a steady five volt output due to the earth's gravitational field. This dc component was balanced by the balancing power supply provided at the input. Using extreme care in balancing the operational amplifiers and the dc signal it was possible to obtain an oscillograph trace of displacement which could be calibrated with that of the LVDT by sketching in the drifting base line. An example of this result is shown in Figure 4.12.

Since the computer used unstabilized amplifiers and was not particularly sophisticated electronically, it was anticipated that these results

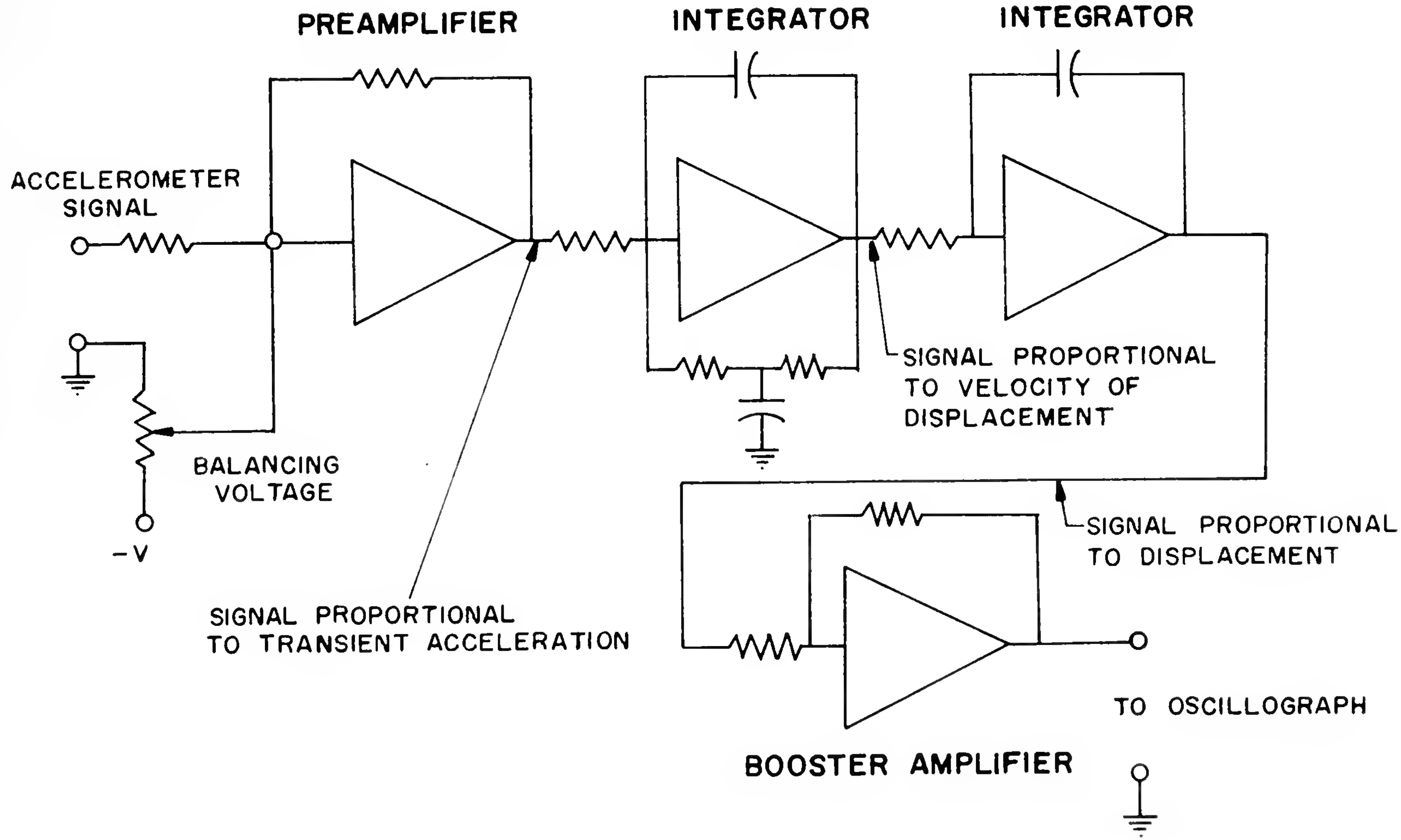


FIGURE 4.11. ANALOG COMPUTER CONFIGURATION FOR DOUBLE INTEGRATION, DIRECT COUPLED

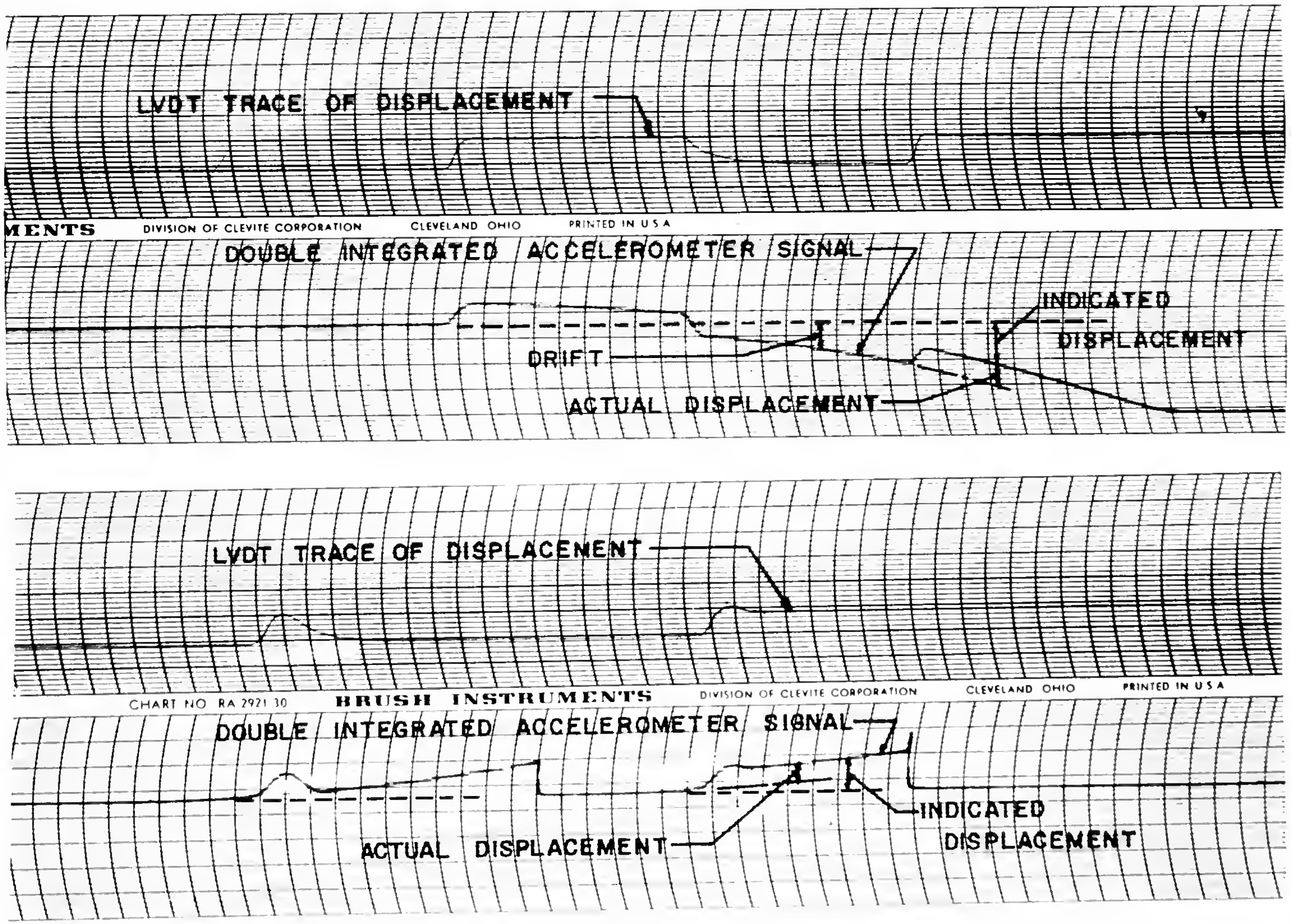


FIGURE 4.12. TYPICAL RECORDS OF DISPLACEMENT AND DOUBLE INTEGRATED ACCELERATION SIGNAL

could be improved significantly by obtaining high quality amplifiers and power supplies incorporating the most recent electronic improvements. To accomplish this a set of Philbrick K2-W amplifiers and a Philbrick R300 power supply were used. These components were assembled and wired in a compact form appropriate for field use. Unfortunately, laboratory tests of this equipment indicated that it was less stable than the computer. Many variations in the equipment were tried in an attempt to reduce the drift to a level which could be tolerated. Filter characteristics were varied, different input and feedback impedance values were tried, and another attempt at ac coupling was made. In no case using direct coupling was it possible to keep the second integral stable enough so the oscillograph record would remain on the chart longer than about one second. Generally, the drift went off scale in much less time. No better success was achieved with ac coupling than had been obtained with the computer.

Two other alternatives were considered to improve the integration process. There was a possibility that solid state amplifiers would be enough more stable than tube amplifiers to achieve the desired results. On the other hand, tape recording field signals and doing the integration on a high quality laboratory analog computer might prove satisfactory. Both of these approaches had several draw backs. Equipment would be considerably more expensive for either approach than had been planned for the project. Field use of transistor circuits had often proved less than satisfactory. No solid state amplifiers were readily available for tests with the laboratory set-up. Tape recording of field data had been tried (21), but this approach leaves field tests uncertain as to success until the tapes are processed in the laboratory. Further time and expense in

pursuing these alternatives without better assurance of success appeared unjustified until field tests of the total concept could be carried out. If full scale trials indicated that the inertial system is feasible for measuring pavement displacements and also that load and displacements are dependent on vehicle velocity, it would certainly be appropriate to refine the electronic integration. Experience with the equipment up to this point, however, had indicated that a single integration could be performed reasonably satisfactorily. Significant drift did occur, but integration could be carried on long enough to record the signal produced by transient pavement displacements, and the drift could be easily established on the oscillograph record to permit establishing a base line for the signal. Operating on the accelerometer signal in this manner produced a record of velocity of pavement displacement on the oscillograph. This record could be integrated manually either by planimeter or by digitalizing the record and performing a numeric integration. While this did not satisfy all of the requirements of the ideal system, it at least promised a workable field installation to get some actual data on full scale pavements.

Three computers were built according to the above plan. The block diagram of Figure 4.13 indicates how three separate channels of displacement data could be processed simultaneously with this equipment. Each channel provided preamplification, one integration, and post amplification of a signal before feeding into the six-channel oscillograph. The post amplification stage was composed of two amplifiers - one a K2-W operational amplifier and one a current boosting amplifier built in the laboratory to provide adequate boost in signal power to drive the oscillograph. Independent gain control was provided for each amplification stage, and separate

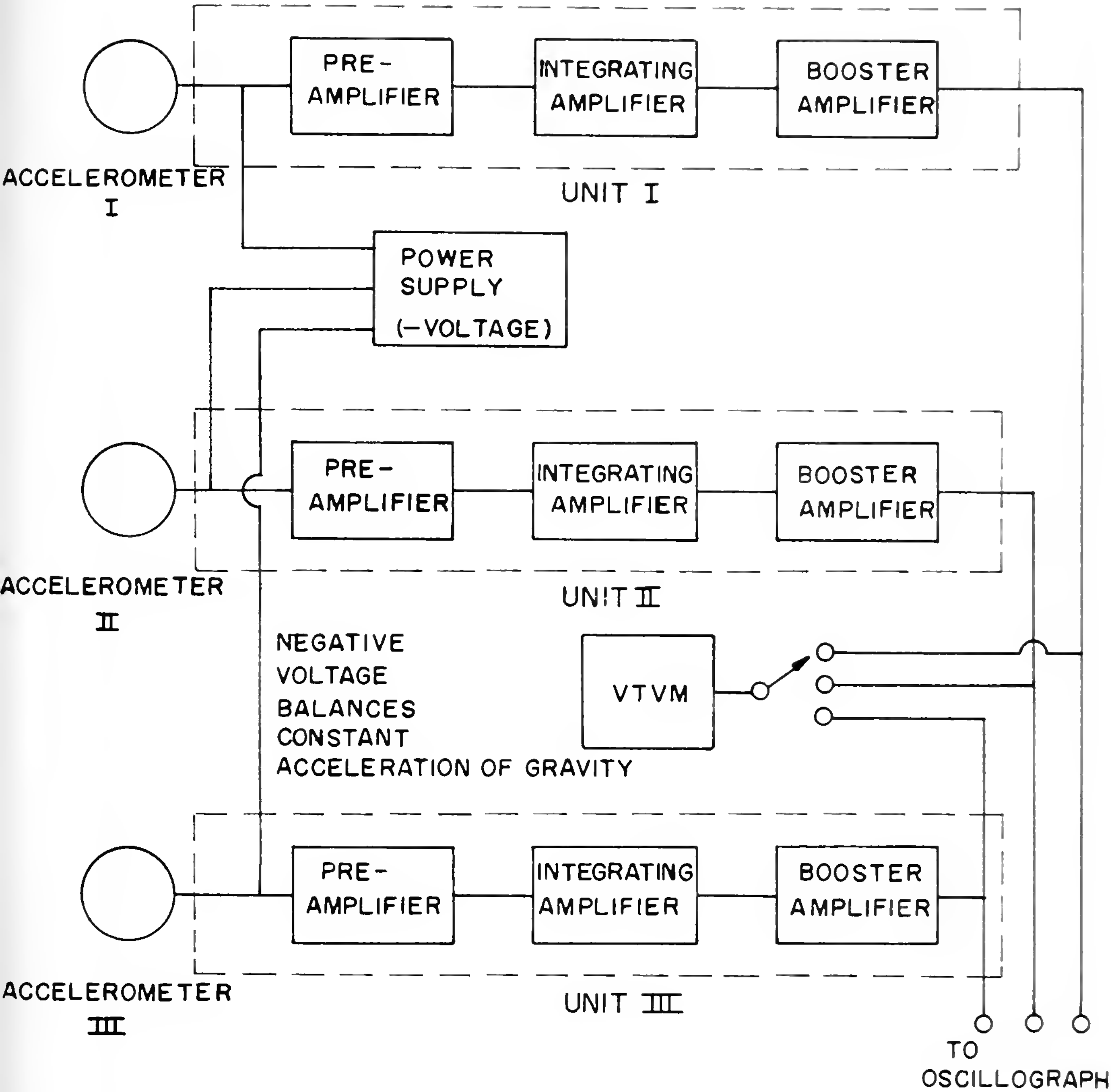


FIGURE 4.13. BLOCK DIAGRAM OF ANALOG COMPUTER USED IN FIELD TESTS

vernier controlled balancing voltage was provided for each channel. Output jacks were provided for monitoring the output with a vacuum tube voltmeter. Switch selection made it possible to observe the output of any stage of any channel with a single meter. The entire assembly could be conveniently mounted in a panel truck for field use.

Calibration

After the transducers were selected and the electronics assembled, calibration of the entire system for measuring displacements was undertaken. This was achieved with the same test table and recording device previously described for initial development.

Basically, the process was to determine the ratio of displacement of the accelerometer to the area under the corresponding curve of velocity. The LVDT trace provided a simple means for determining displacement in any time interval. To correspond to anticipated field conditions, the displacements used in calibrating were restricted to transient pulses. Initially, the area under the velocity curve for the same interval was obtained by planimeter. This process was extremely painstaking, however, as the areas in many cases were quite small. It proved possible to get more consistent results by digitalizing the velocity record and computing the area by numeric integration. Even with this procedure there was considerable variation in the displacement to area ratio. This error appeared to be associated primarily with the oscillograph. There was a significant variation in frequency response of the oscillograph, and more serious, a delay in the pen returning to zero when the signal was removed. This behavior suggested improper damping of the pen motor, and a considerable

effort was spent in trying to correct it, but no solution to the difficulty was found. An additional source of error apparently was friction of the pen on the chart paper. All of these effects, as well as errors in the integration, apparently contributed to the variation of the calibration. It was necessary, therefore, to use a statistical calibration. A typical example of calibration data is shown in Figure 4.14. Since the accelerometer, amplifiers and oscillograph were all theoretically linear, a straight line calibration was expected and appears reasonable from these data. The line drawn through the observed points was obtained by a least squares fit of a straight line passing through the origin. The very high correlation coefficient (in this case, 0.99725) appears to verify the linear functional relationship. The standard error of estimate was 0.00376 for this example, and was generally of this order of magnitude.

In order to obtain some idea of the precision of measurements obtainable, confidence limits were plotted for the estimated mean displacement. The lines plotted above and below the regression line in Figure 4.14 indicate these limits in the illustrated case. One departure from usual statistical practice should be noted in this connection. Because it was impossible in the calibration procedure to observe displacements for arbitrary values of area under the velocity curves, the calibration was fitted using displacement as the independent variable. Statistically, this provides an equation

$$\text{area} = m \times \text{displacement} \qquad 4.9$$

based on minimum squared variations of observed areas about the regression line. The confidence interval is therefore evaluated for area

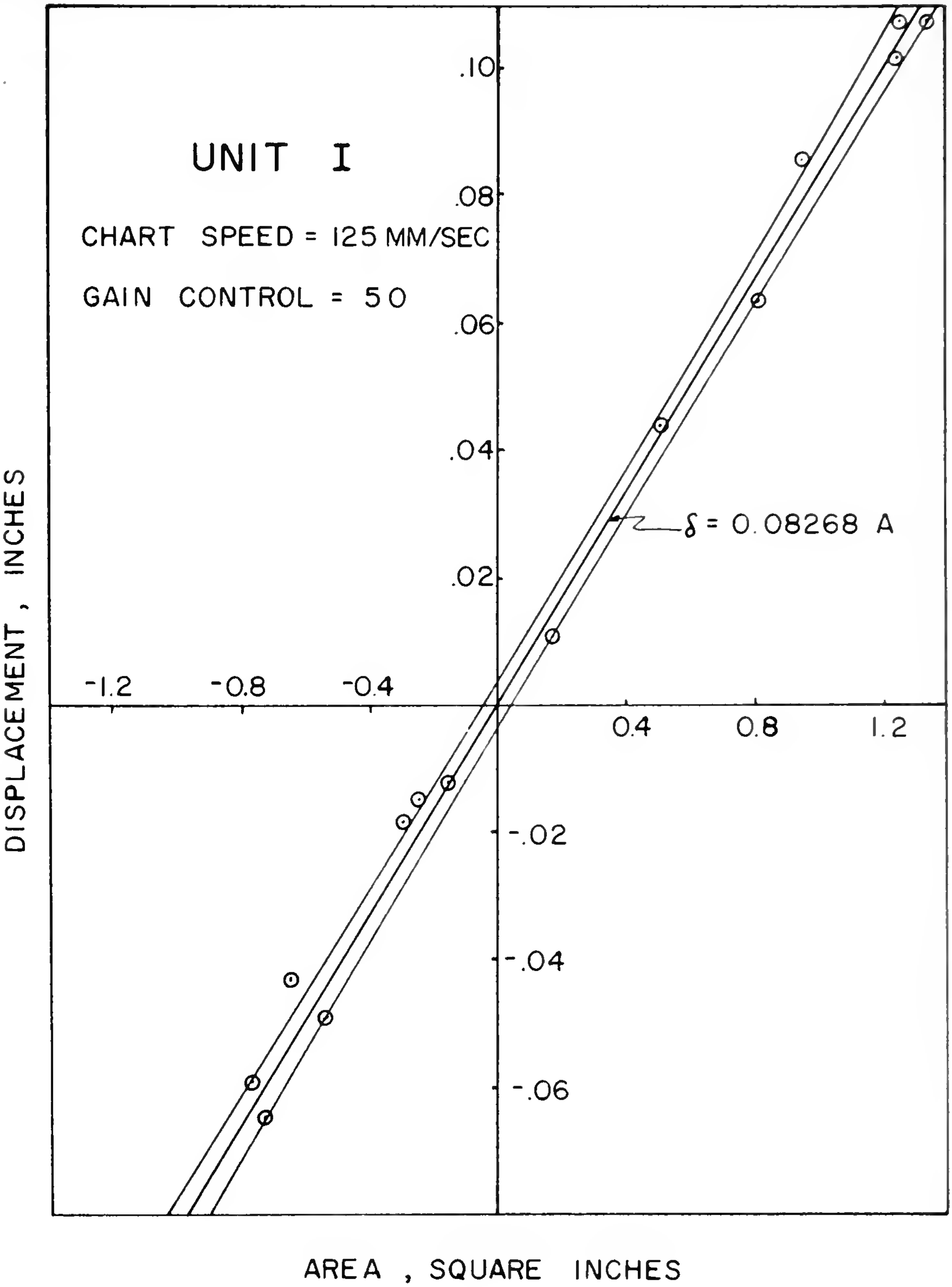


FIGURE 4.14. TYPICAL CALIBRATION OF DISPLACEMENT MEASURING SYSTEM

obtained for a given displacement. An estimate of mean displacement about the regression line is obtained by reading the confidence interval in the coordinate direction of displacement at a given area. The confidence interval shown in Figure 4.14 is typical and indicates a variation at the 5 percent level on the order of two thousandths of an inch. It would be desirable if this variation could be reduced to less than one thousandth, but the present results were considered satisfactory for the initial field studies.

The interpretation of field records of displacement consisted of reading ordinates to the velocity curve, integrating numerically, and applying the calibration factor 1/m from equation 4.9. Simpson's rule was used for the integration, and, because of the large volume of data to be resolved, the work was done on an IBM 7094 digital computer. The 7094 program provided for a listing of displacement versus vehicle position at discrete intervals and a plot of these data for each run. Figure 4.15 shows a typical field oscillograph record containing traces of both stress and velocity of displacement versus time. Figure 4.16 illustrates the digital computer output consisting of the transformed displacement data corresponding to the surface velocity trace of Figure 4.15.

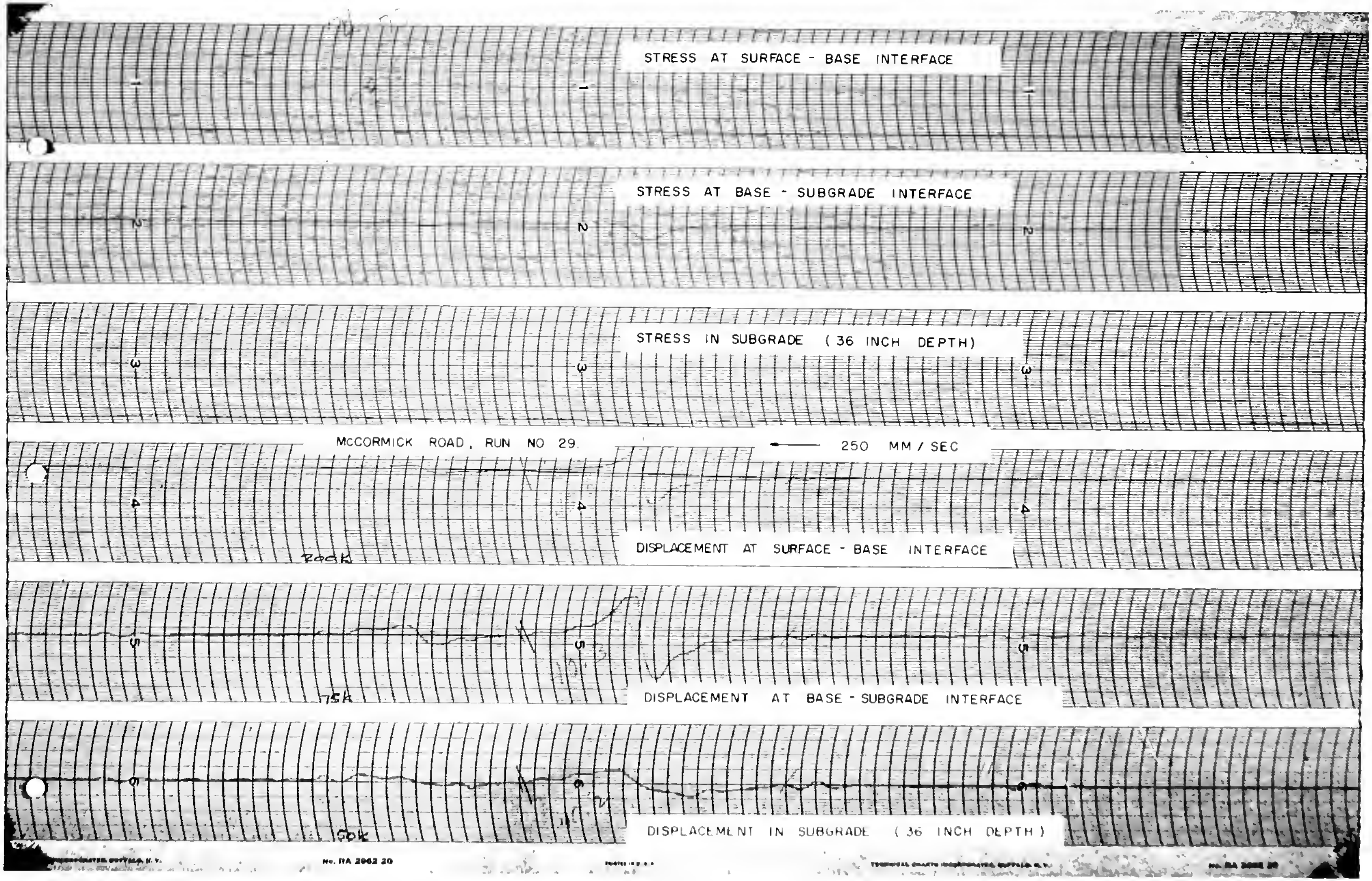


FIGURE 4.15. TYPICAL FIELD RECORDS OF STRESS AND DISPLACEMENT

C-36-52-F DEFLECTION DATA 1964
MCCORMICK ROAD
SURFACE DEFLECTION
TEST NO. 29
VEHICLE VELOCITY= 30.15 MPH

CHART SPEED= 250.00 UNITS/SEC   CAL= .17400   UNITS= 25.40   H= 5.0000
INCREMENT OF DISTANCE= 1.76871

I	Y(I)	DISTANCE (FT.)	DEFLECTION (IN.)	INDEX
1		-7.075	.00000	1
3	.80	-5.306	.00162	2
5	1.70	-3.537	.00454	3
7	3.30	-1.769	.01038	4
NOTE CHANGE OF INTERVAL-INCREMENT OF DISTANCE= .88435				
3	7.70	-.884	.01735	5
5	.50	-.000	.02998	6
7	-10.00	.884	.01949	7
9	-5.30	1.769	.00903	8
NOTE CHANGE OF INTERVAL-INCREMENT OF DISTANCE=1.76871				
3	-2.20	3.537	-.00081	9
5	-1.30	5.306	-.00562	10
7	-.90	7.075	-.00841	11
9	-.[illegible]	8.844	-.00971	12

C-36-52-F DEFLECTION DATA 1964
MCCORMICK ROAD
SURFACE DEFLECTION
TEST NO. 29
VEHICLE VELOCITY= 30.15 MPH

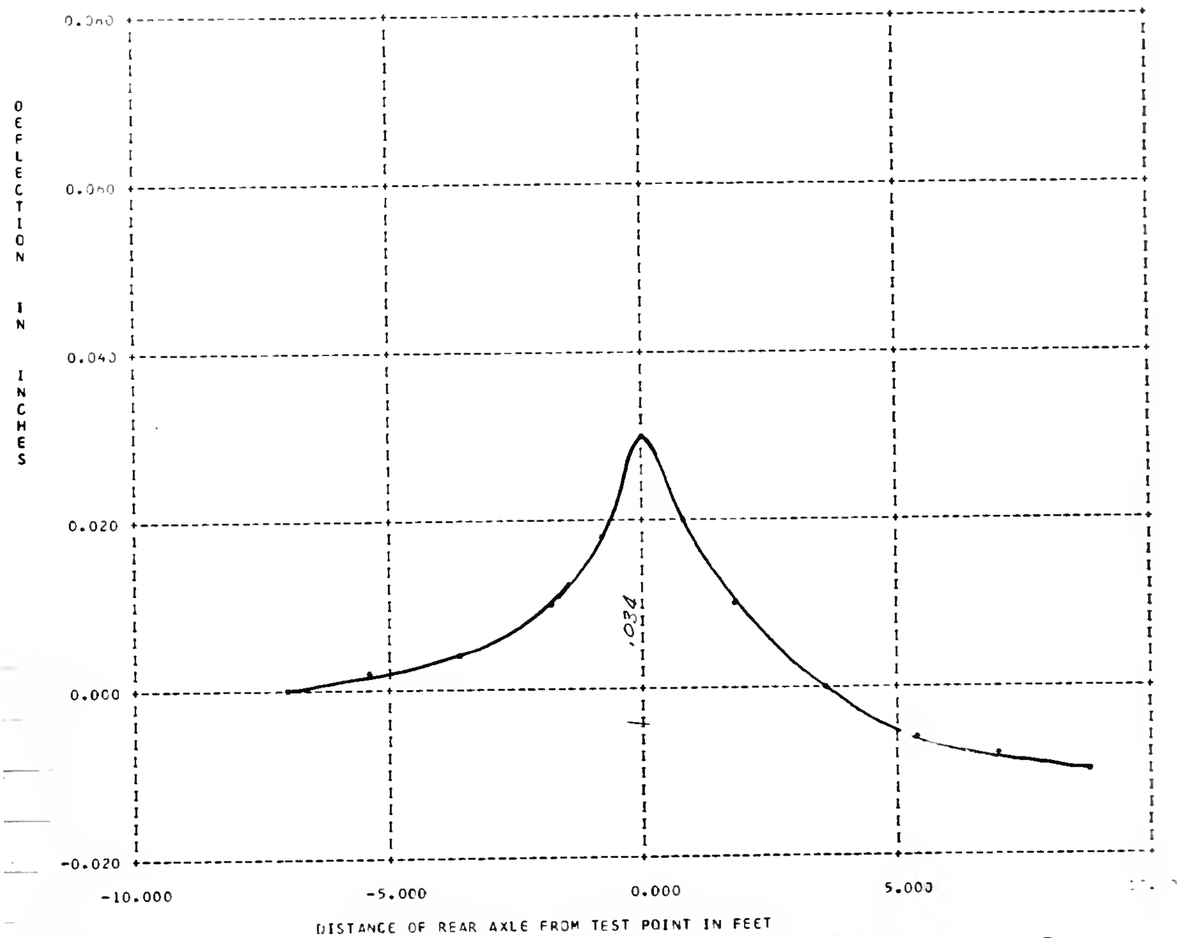


FIGURE 4.16. TYPICAL DISPLACEMENT VS LONGITUDINAL POSITION OF TRUCK

5. FIELD TESTING

Field tests of the capability of the tire pressure measuring equipment for passenger cars were successfully carried out during the summer of 1963 (48). During the following winter, the basic modifications of the system to suit heavy vehicles were made. In the same period transducers and electronic equipment for measuring stresses and displacements in the pavement were assembled. Within the limitations described in previous sections, the total system was workable and it was determined to run an initial series of field tests on several pavements of different physical characteristics in order to evaluate the system and to obtain some indication of the significance of the parameters in question. The tire pressure equipment was mounted on a truck and flexible pavement sections in the vicinity of Lafayette were selected. Tests were run during August and September of 1964 and data analysis proceeded for several months following.

Instrumentation

Test Vehicle

Through the cooperation of the Indiana State Highway Commission, a dump truck was obtained for the duration of the field testing period. The truck had a single rear axle equipped with dual wheels. Tires were 10.00-20/12 having a rated load capacity of 4600 lbs at an inflation pressure of 75 psi. A governor on the engine limited vehicle velocity to a maximum of about 60 mph. The truck is pictured in Figure 5.1.

FIGURE 5.1. THE TEST TRUCK

Wheel Loads

The tire pressure measuring equipment was mounted on the left hand side at the rear axle. The left hand tires were selected for measurement because it was desired to measure pavement behavior near the center of the pavement structure. This would eliminate effects of the pavement edges. The rotating seal was mounted on the outer dual and the two parts connected to the stems of the two tires. The instrument panel was bolted to the body immediately above the axle and pressure lines from the rotating seal connected to the panel. The final field setup is shown in Figure 3.2 The amplifiers and oscillograph were mounted in a specially constructed box which replaced the seat on the passenger side of the cab. A motor generator set to provide power for the electronic equipment was supported on a platform fastened to the snow plow attachment at the front of the vehicle.

Event markers consisting of 3/4 inch diameter sections of pipe were placed on the pavement in the path of the instrumented wheel. When the instrumented wheel hit a pipe, a sharp pulse which was readily identified was produced on the oscillograph record and served to relate the vehicle displacement and time scales. On the first installation at McCormick Road, the pipes were placed only six feet on either side of the instrumented point. However, the disturbance to the pressure measuring system as the vehicle passed over the pipe was noticeable for several feet beyond that point, and a vibratory effect appeared in the pavement displacement record which seemed to result from the same impact. For subsequent tests, the pipes were moved to 20 feet either side of the instrumented point, effectively removing the disturbance from the region of interest.

Stresses and Displacements

Transducers and amplifiers and recording equipment for measuring stresses and displacements within the pavement and subgrade were described in detail in Part 4. The stress cells and accelerometers were assembled and wired with ample lead cables prior to field installation. In order to obtain the desired objective of minimal disturbance of the test structure, it would be best to install the transducers in the desired positions in a test pavement during construction, with necessary cables being carried horizontally through the pavement to the outer edge. For the preliminary series, however, since in-service pavements were to be used, it was necessary to install the instruments from the surface.

Installation. Placement of transducers from the pavement surface required that a hole be bored to the greatest depth at which measurements would be obtained. A six inch square opening was cut through the bituminous surface and the aggregate layers with hand tools. A four inch diameter hand auger hole was then advanced to the desired depth. At the bottom of the hole a transducer was held in a vertical position at the precise horizontal location by a specially built jig attached to a length of rigid galvanized pipe. The excavated soil, which had been carefully saved, was then replaced. As soon as enough soil was tamped around the transducer to hold it firmly, the jig was removed. A length of copper tubing inside the pipe and jig held the transducer in place as the jig was pulled off. Backfill was continued to the depth of the next transducer, where the imbedment procedure was repeated. Compaction of the backfill was accomplished by tamping and was intended to reproduce as nearly as possible the undisturbed condition of the pavement and subgrade. After all transducers were in

place and backfill was complete, the hole in the bituminous surface was filled with a patching mix of bituminous concrete.

Cable leads had been brought to the surface along the edge of the hole. Lengths of rubber hose were split and installed over the cables at the point where they emerged from the pavement to give additional protection to the wires. The cables were then run to the edge of the pavement and taped to the surface.

Two Controflex RB-5 Road Switches were placed transversely across the pavement, one ten feet ahead, and the other ten feet following the instrumented point. These switches activated an event marker on the oscillograph each time a wheel passed over them, providing a means of determining vehicle velocity and relating longitudinal vehicle displacement to the oscillograph time scale. A typical installation is illustrated in Figure 5.2.

Operation

The test procedure used for the entire series was to establish initial conditions of all instruments with the truck standing some distance from the instrumented point, then to drive the truck past the test point at a specified speed and lateral position. The recorders were operated to give a continuous record of tire pressure variation, soil stresses and displacements for a time interval during which the instrumented points in the pavement were influenced by the load. For low speed tests, the initial point of the run was quite close to the test point, on the order of 50 feet. As the vehicle speed was increased the distance to the starting point was increased. For the fastest test runs, near 60 mph, it was necessary to begin the run from nearly one mile away from the test point.

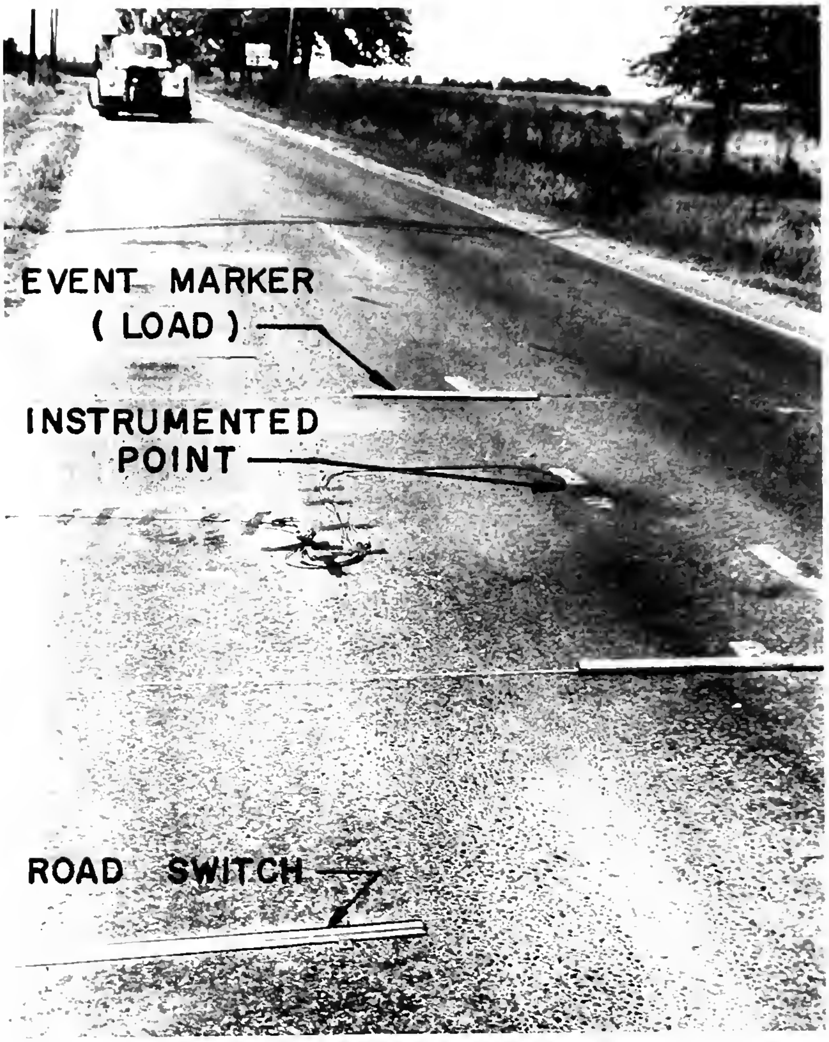


FIGURE 5.2. TYPICAL PAVEMENT TEST INSTALLATION

In order to provide exactly the same initial tire pressure (and hence load) conditions for all runs of a given speed, the position of the wheels was marked on the pavement at each starting location, and the truck positioned precisely for each run. Relative elevations of the four wheels were obtained to provide information for measuring the exact tire load in each initial condition. This calibration did not work successfully, however, so that all pressure variations were ultimately superimposed on the static load of the tires for the level condition.

Initialization of the displacement measuring system involved balancing all operational amplifiers and adjusting the input balancing voltage to zero signal for the static condition. The computers were placed in operation well before the truck approached the instrumented point, and the output monitored on a voltmeter. Major drift was counteracted by adjusting the signal balancing circuit during operation. As the truck approached the first road switch, the oscillograph was turned on and the computer output was switched to the oscillograph. The stress measuring equipment was comparatively stable and required only occasional checks on the excitation voltage and amplifier gain.

Records

The oscillograph records (two channels in the test truck and six in the instrument van) provided continuous traces of the measured parameters with respect to time. Event marks on the charts permitted evaluation of the exact vehicle speed for each test run and also identified fixed locations on the charts. With location and vehicle velocity both known, the relation between the chart time scale and vehicle displacement was defined.

The two channel chart in the truck gave independent records of pressure variations of the left rear inner and outer dual tires. These records would give a qualitative indication of variations in tire force. However, the force to pressure ratio was so greatly dependent on frequency of variation that direct quantitative interpretation was not possible. The quantitative analysis was described in Part 3.

Six channels of data were obtained in the instrument van from three combined stress-displacement transducers in the pavement. Three channels of stress data gave directly, with the previously obtained calibration curves, the dynamic soil stresses induced at the transducers when the test vehicle moved over the location. Three channels of displacement data were actually traces of the velocity of movement of the transducers. It was possible to make gross judgments from the raw data as to whether adequate response was being obtained on each run, but because the displacement was represented by the area under the velocity curve, no estimate was made in the field of magnitudes of displacements. It was possible to observe from the raw data the radius of influence of the wheel load and to make qualitative comparisons between fairly smooth, continuous displacements and rough, jerky displacements. The latter appeared to occur when the tire loads were rough as a result of sharp impact as the vehicle approached the test point.

Quantitative evaluations of dynamic displacement were made as described in Part 4. The field records of velocity of displacement were digitalized and integrated numerically. The resulting displacements at discrete intervals were tabulated and plotted by digital computer in the form of displacement versus distance of rear axle (the instrumented tires) from the test

point in the pavement. Smooth curves were drawn free hand through the plotted points to aid in visualizing the displacement pattern.

Test Conditions

For the series of tests run in 1964, the test truck was loaded to produce a static weight of 18,000 lbs. on the rear axle - 4,500 lbs on each tire. Nominal tire pressure was maintained at 75 psi throughout the series.

The three pavements tested were located on McCormick Road, 1/4 mile south of Cherry Lane at the western edge of West Lafayette, Indiana Road 18, one mile west of Brookston, and Indiana Road 26, 4-1/2 miles west of West Lafayette. These locations are shown on the map of Figure 1.1.

The McCormick Road pavement consisted of four inches of bituminous concrete and eight inches of macadam base on a subgrade of low plasticity clay. The Road 18 site was a 5.5 inch bituminous concrete surface and a ten inch gravelly, clayey sand base course on a low plasticity clay subgrade. At Road 26, the pavement surface was seven inches of bituminous concrete and the base was 13 inches of gravelly, sandy clay. The subgrade was a borderline silt-clay of low plasticity. Table 5.1 summarizes the physical characteristics of the three pavements. It should be noted that the pavements were selected with the intention of having a wide range of stiffnesses represented in the pavement structures, but as nearly as possible the same subgrades and construction types. Preliminary investigations of the sites at the pavement edges indicated similar materials throughout. It was only during installation of transducers that the base course at McCormick Road was found to be of macadam. The subgrade at

TABLE 5.1

SUMMARY OF PAVEMENT CHARACTERISTICS

Characteristic \ Site	McCormick Road	Indiana Road 18	Indiana Road 26
Bituminous Surface			
Thickness (in.)	4	5.5	7
Base Course			
Type	Macadam	Sand-Gravel	Sand-Gravel
Thickness (in.)	8	10.0	13
Classification[1]		SC	SC
Field Density[2] (pcf)	Not Determined	135.0	127.8
Field Moisture (%)	6.5	4.3	7.9
Liquid Limit[3] (%)	25.3	26.0	22.0
Plasticity Index[4] (%)	8.2	8.7	5.4
Optimum Density[5] (pcf)	Not Determined	136	130
Optimum Moisture[5] (%)	Not Determined	8	9
Subgrade			
Classification[1]	CL	CL	CL-ML
Field Density[2] (pcf)	111.7	113.7	101[7]
Field Moisture (%)	17.5	16.8	23.7
Liquid Limit[3] (%)	28.4	35.5	40.6
Plasticity Index[4] (%)	8.9	13.7	14.8
Optimum Density[6] (pcf)	107.9	111.0	103.4
Optimum Moisture[6] (%)	14.8	12.4	17.8

Notes:
1. According to Unified Soil Classification System
2. AASHO T 147-54
3. AASHO T 89-60
4. AASHO T 91-54
5. AASHO T 99-57, Method D
6. AASHO T 99-57, Method A
7. Estimated Value - Field test was in error

Road 18 also was basically like the other two, though it had a slight organic content which probably accounts for the silty characteristic.

At each site transducers were placed at the surface-base interface, the base-subgrade interface and in the subgrade at a depth of 36 inches from the pavement surface.

Tests were run at nominal speeds of 5, 10, 20, 30, 40, 50 and 60 mph and nominal transverse offsets of 0, 6 and 12 inches. The offset was measured as the horizontal distance from the vertical line of the transducers to the center line of the path of the dual wheels in a direction normal to the center line of the pavement. Distances to the right facing the direction of travel are positive, and to the left, negative. In order to obtain this measurement, a stripe was painted on the pavement just prior to each run. The tire imprint was clearly visible in the fresh paint. It was attempted to make at least two runs at each velocity and each offset. In practice, it was not possible to obtain close control over either of these variables. Speeds varied by several miles per hour from nominal, mostly below nominal, although control and consistency were much better at Road 18 and Road 26 than at McCormick Road. This was probably due both to experience of the driver and better operating conditions at those sites. Variation in offset was even greater than in velocity, and measured values of offset ranged from zero to sixteen inches in absolute value. In view of this poor control, a tolerance of plus or minus two inches was adopted with respect to obtaining the desired lateral position. Data was obtained from all runs, regardless of offset and velocity, and ultimately the actual values of these parameters were used in the analyses.

Following testing at each site, the stress-displacement transducers were removed for reuse. Tests of in-place moisture and density were made on the base course and subgrade. Samples of each material were taken for laboratory classification. The results of these tests are included in Table 5.1.

6. RESULTS

The series of tests in the summer of 1964 resulted in a total of 142 runs. At the McCormick Road site, 36 runs were made ranging in velocity from three to 40 miles per hour. At Indiana Road 18, 50 runs ranged from four to 58 miles per hour, and at Indiana Road 26, 52 runs ranged from three to 59 miles per hour. Table 6.1 presents a summary of the test runs, indicating for each run the lateral position of the vehicle, vehicle velocity, inner and outer wheel load at the test point, and displacements and stresses at the transducer locations in each pavement. Maximum speed was determined by a governor on the test truck set for a nominal 60 miles per hour. At McCormick Road it was not possible to reach this limit because of rough surface and curved alignment about 1/4 mile from the test point. Runs 15 through 18 for Road 18 are omitted from Table 6.1. These runs were an attempt to obtain measurements for an essentially static load, but no usable results were obtained.

Loads

Records of dynamic tire pressure were read for an interval corresponding to approximately 40 feet of road - that distance between event markers on all but the first day's runs. These records were transformed to digital force records indicating both the dynamic force and the actual force each tire was exerting on the pavement at closely spaced discrete points. The actual force at each point represents the sum of the dynamic force and the initial static load of 4,500 pounds. A plot of actual force

Table 6.1

SUMMARY OF DATA

SITE	RUN	OFFSET (IN)	VEHICLE VELOCITY (MPH)	LOAD		DISPLACEMENT			STRESS		
				OUTER DUAL (KIPS)	INNER DUAL (KIPS)	SURFACE (IN)	BASE (IN)	SUBGRADE (IN)	SURFACE (PSI)	BASE (PSI)	SUBGRADE (PSI)
1*	1	1.00	3.02	4.89	4.39	.023	.010			4.71	.94
1	2	6.00	4.96	4.42	3.71	.031	.021			5.80	
1	3	5.75	3.90	4.71	4.02	.030	.023			5.55	2.44
1	4	1.75	4.18	4.54	4.04	.026	.020			5.69	.85
1	5	1.00	9.56			.027	.020			4.06	1.45
1	6	3.00	7.91	4.86	4.18	.033	.026			4.55	1.37
1	7	-.50	8.82	4.48	3.69	.031	.025	.007		4.22	1.37
1	8	-6.50	7.83	4.76	3.62	.043	.026	.008		6.58	1.62
1	9	-10.00	9.49	4.55	3.65	.029	.021	.006		5.12	1.19
1	10	-5.50	7.68	4.83	4.55	.045	.025			6.74	1.71
1	11	-12.50	8.82	4.36	4.14	.023	.019	.006		4.06	1.02
1	12	-5.75	14.88	4.54	3.83					6.65	1.79
1	13	-2.00	16.90	4.59	4.29	.042	.025	.008		4.71	1.62
1	14	-1.50	17.20	4.42	3.88	.034	.024	.005		4.31	1.45
1	15	-16.50	18.00	4.06	4.51	.014	.013	.003		1.71	1.06
1	16	-5.25	15.45	4.21	4.77	.044	.026	.007		6.01	1.54
1	17	-15.25	15.60	4.17	4.86	.016	.016	.007		2.19	.93
1	18	-6.75	15.30			.043	.023	.008		5.93	.93
1	19	-2.25	20.90	4.19	3.42	.042	.022	.005		4.06	.93
1	20	-2.00	22.01			.041	.026	.006		4.39	1.37
1	21	-2.75	20.81	4.35	3.34	.039	.022	.007		3.90	1.02
1	22	-14.50	23.80	4.58	4.88	.020	.019	.007		2.60	.93
1	23	-16.50	25.74	4.21	3.96	.015	.012	.006		1.71	1.22
1	24	-9.75	26.10	4.82	4.50	.033	.022	.006		4.71	1.54
1	25	-2.00	26.54	3.95	4.27	.039	.021	.006		3.57	1.19
1	26	1.25	23.20			.034	.026	.006		3.90	1.45
1	27	-.50	35.51	4.45	5.16	.026	.021	.005		3.17	1.19
1	28	-1.00	31.25	4.48	4.29	.031	.022	.006		3.98	1.71
1	29	-9.00	30.15	4.32	4.57	.034	.019	.006		4.31	1.19
1	30	-12.50	34.81	4.63	4.51	.021	.015	.006		2.92	1.11
1	31	-12.00	35.34			.021	.016	.006		3.25	1.02
1	32	-10.50	34.90	5.04	4.41	.025	.018	.006		3.25	1.11
1	33	-3.75	36.03				.022	.007		4.31	1.62
1	34	-3.00	36.24	4.79	3.96	.034	.022	.007		4.06	1.28
1	35	-7.50	35.12	4.55	4.02		.020	.006		4.31	1.45
1	36	-.00	39.50	4.64	3.99	.032	.024	.007		3.66	1.19

* MC CORMICK ROAD

Table 6.1 (Continued)

SUMMARY OF DATA

SITE	RUN	OFFSET (IN)	VEHICLE VELOCITY (MPH)	LOAD: OUTER DUAL (KIPS)	LOAD: INNER DUAL (KIPS)	DISPLACEMENT: SURFACE (IN)	DISPLACEMENT: BASE (IN)	DISPLACEMENT: SUBGRADE (IN)	STRESS: SURFACE (PSI)	STRESS: BASE (PSI)	STRESS: SUBGRADE (PSI)
2*	1	5.50	4.71	4.52	4.11	.059	.035	.005		11.54	
2	2	2.50	5.62	4.30	3.64	.064	.043	.005		10.48	
2	3	-3.00	4.60	4.47	4.37	.082	.052	.007		11.2[illegible]	
2	4	[illegible].75	3.66	4.39	5.15	.068	.042			12.19	
2	5	10.50	4.04			.042	.023			6.74	
2	6	7.00	5.09	4.48	4.51	.053	.029	.005		9.91	
2	7	15.00	5.99	4.41	4.74	.021	.017	.003		1.62	
2	8	2.00	5.39	4.30	5.31	.068	.044			10.89	
2	9	3.00	5.92	4.58	5.51	.066	.043	.005		11. 5	
2	10	.00	5.87	4.10	4.71	.047	.039	.006		10.48	
2	11	9.50	5.76	4.11	5.01	.046	.026	.004		5.61	
2	12	4.25	8.40	4.34	5.15	.062	.035	.006		10.64	
2	13	12.75	9.32	4.25	4.58	.034	.019	.005		3.25	
2	14	10.50	8.61	4.27	4.63	.040	.030	.005		5.12	
2	19	.00	17.54			.041	.034	.005		7.47	
2	20	.00	17.59	4.40	5.40	.014	.031	.006		5.12	
2	21	-2.50	17.54	4.30	5.56	.039	.029	.006		6.[illegible]6	
2	22	8.25	17.45	4.18	4.53	.027	.019	.005		3.17	
2	23	[illegible].25	17.61	4.24	5.02	.026	.021	.005		2.76	
2	24	11.75	17.53	4.03	4.17		.018	.004		1.71	
2	25	11.75	17.61	4.44	4.65		.017	.004		1.54	
2	26	1.25	26.16	4.63	5.69		.040	.007		5.69	
2	27	2.75	30.56	4.35	5.69		.035	.005		4.[illegible]7	
2	28	-7.75	30.86	4.68	5.92		.039	.006		7.[illegible]0	
2	29	.25	30.78	4.50	5.73	.050	.039			4.57	
2	30	10.75	30.56	4.85	5.65	.033	.020			1.79	
2	31	7.25	30.78	4.57	6.70	.045	.031			4.[illegible]	
2	32	7.75	30.69	4.51	7.26	.044	.032			4.22	
2	33	10.00	30.52	4.61	4.70	.033	.020			1.57	
2	34	-1.00	32.30	4.34	5.71	.054	.037			[illegible]	
2	35	-.25	36.39	4.31	4.56	.048	.036			5.[illegible]1	
2	36	-3.00	36.33	4.29	4.36	.056	.039			7.[illegible]8	
2	37	10.25	37.24	4.32	4.33	.029	.019			[illegible]	
2	38	5.75	37.24	4.3[illegible]	4.57	.040	.026			[illegible]	
2	39	4.75	37.24	4.33	6.17	.039	.028			[illegible]	
2	40	10.25	37.56	4.67	4.26	.032	.020			[illegible]	
2	41	-.50	40.14	4.51	4.66	.051	.035			[illegible]	
2	42	-2.75	43.62	4.71	4.94	.055	.039			7.3[illegible]	
2	43	-2.00	42.94	4.52	5.03	.049	.034			[illegible]	
2	44	[illegible].75	48.17	4.65	5.51	.035	.029			[illegible]	
2	45	8.50	45.68	5.11	6.31	.042	.032			[illegible]	
2	46	[illegible].00	50.25	5.22	6.35	.038	.032			[illegible]	
2	47	12.50	45.68	5.13	5.24	.029	.023			1.[illegible]	
2	48	10.50	45.68	4.57	6.08	.034	.027			1.71	
2	49	1.00	57.18	4.86	7.37	.040				[illegible]	
2	50	.75	58.17	[illegible].24	8.37		.042			[illegible]	
2	51	11.75	58.31	5.02	5.05	.034	.020			1.[illegible]	
2	52	7.50	58.31	4.53	7.31		.026			[illegible]	
2	53	5.50	58.16	4.54	7.12		.032			4.7[illegible]	
2	54	12.00	58.31	4.78	7.18	.024	.020			1.6[illegible]	

* INDIANA ROAD 18

Table 6.1 (Continued)

SUMMARY OF DATA

SITE	RUN	OFFSET (IN)	VEHICLE VELOCITY (MPH)	LOAD: OUTER DUAL (KIPS)	LOAD: INNER DUAL (KIPS)	DISPLACEMENT: SURFACE (IN)	DISPLACEMENT: BASE (IN)	DISPLACEMENT: SUBGRADE (IN)	STRESS: SURFACE (PSI)	STRESS: BASE (PSI)	STRESS: SUBGRADE (PSI)
3 *	1	-4.25	4.07	4.63	4.66	.024	.014			2.56	1.87
3	2	-.25	3.30	4.68	4.97	.020	.012			2.44	2.13
3	3	-1.25	4.96	4.52	4.67	.021	.016			2.36	1.96
3	4	7.25	5.65	4.47	5.13	.020	.010			1.79	1.54
3	5	5.25	5.89	4.05	5.26	.019	.011			1.71	1.54
3	6	10.00	6.17	4.47	4.82	.014	.009			2.03	1.54
3	7	9.75	5.15	4.69	4.95	.013	.010			2.19	1.62
3	8	-.75	8.59	4.40	4.97	.021	.018			2.03	1.71
3	9	-.25	6.51	4.30	4.87	.021	.012			2.52	1.71
3	10	6.00	9.74	4.09	5.18	.019	.011			2.11	1.71
3	11	5.25	6.22	4.58	6.13	.020	.006			2.36	1.62
3	12	10.00	8.79	5.03	4.56	.014	.008			1.79	1.62
3	13	11.25	7.32	5.41	4.61	.014	.009			2.03	1.19
3	14	-2.00	17.66	4.28	4.60	.023	.017	.006		1.87	1.71
3	15	1.75	17.45	4.04	4.65	.020	.016	.005		2.11	1.71
3	16	-4.75	17.59	4.47	3.83	.027	.016	.006		2.56	1.71
3	17	-.75	17.73	4.69	5.07	.023	.017	.006		2.44	1.71
3	18	6.25	17.59	4.31	5.10	.023	.017	.006		2.03	1.62
3	19	5.75	17.92	5.02	5.98	.024	.016	.007		1.79	1.71
3	20	11.50	17.59	5.40	4.63	.017	.013	.006		1.71	1.54
3	21	10.00	17.56	5.05	4.66	.017	.013	.006		1.62	1.54
3	22	12.75	17.90	5.10	4.21	.016	.013	.006		1.71	1.54
3	23	.50	20.52	4.12	4.66	.021	.017	.006		1.87	1.45
3	24	-.50	30.29	4.45	4.94	.021	.017	.007		2.11	1.71
3	25	-2.25	29.58	4.38	5.16	.027	.018	.007		2.27	1.71
3	26	9.00	28.78	4.57	4.77	.018	.016	.006		1.62	1.54
3	27	7.25	27.96	4.53	4.41	.023	.017	.007		2.27	1.45
3	28	4.75	29.23	4.45	5.60	.021	.015	.007		2.19	1.96
3	29	8.25	29.16	4.58	3.86	.018	.013	.006		1.62	1.11
3	30	11.75	29.25	4.65	4.09	.017	.014	.005		1.62	1.45
3	31	12.25	29.84	5.01	4.13	.014	.013	.006		1.71	1.45
3	32	1.00	38.07	4.87	5.28	.022	.018	.008		2.19	2.04
3	33	1.00	39.08	5.10	5.53	.022	.016	.006		2.36	1.71
3	34	8.75	38.67	5.10	4.93	.019	.014	.006		1.62	1.75
3	35	6.00	38.74	4.75	4.99	.024	.014	.006		1.87	1.62
3	36	7.00	39.22	4.75	4.71	.022	.014	.006		1.71	1.37
3	37	10.75	39.22	5.01	4.66	.020	.014	.006		1.71	1.62
3	38	14.75	39.82	5.74	4.53	.015	.013	.006		1.54	1.02
3	39	8.25	39.15	5.29	5.41	.024	.016	.007		1.87	1.71
3	40	-4.50	48.54	5.50	5.26	.029	.020	.009		2.56	2.19
3	41	2.00	46.41	5.35	5.91	.021	.016	.007		1.95	1.71
3	42	4.00	49.02	4.97	5.55	.029	.017	.007		2.03	1.62
3	43	-3.00	49.91	5.07	5.67	.029	.018	.008		2.36	1.79
3	44	11.50	50.13	5.36	4.71	.019	.014	.007		1.62	1.37
3	45	10.50	48.07	5.40	4.83	.020	.015	.006		1.54	1.37
3	46	1.25	55.40	4.63	5.86	.022	.016	.007		1.79	1.62
3	47	-3.00	54.89	5.23	5.83	.028	.018	.008		2.19	1.22
3	48	1.50	57.56	5.02	5.83	.023	.017	.007		2.27	1.71
3	49	8.50	57.56	5.44	5.57		.015	.007		1.95	1.54
3	50	7.50	58.31	5.30	5.37	.017	.016			2.03	1.54
3	51	10.25	57.93	5.36	4.57	.020	.014			1.71	1.28
3	52	12.00	58.86	5.14	4.43	.015	.011			1.30	1.71

* INDIANA ROAD 26

for each tire versus position of the rear axle with respect to the test point was also produced for each run. A typical curve is illustrated in Figure 3.4. The load shown in the summary of Table 6.1, and used in the subsequent analysis, was taken as the maximum value for each tire occurring in an interval of 12 inches centered at the test point. This corresponds to the time interval during which the tire made contact with the pavement on the vertical axis of the transducers. For analyses involving the total dual wheel load, the sum of the two tire loads from Table 6.1 was used.

On a few runs, notably on McCormick Road, the oscillograph records were unreadable. This resulted from excessive vibrations which caused the pen to jump off the paper or from loosened electrical connections. The smoother pavements at the other two sites, and some additional vibration damping on the equipment reduced this problem.

Stress and Displacement

The stress values indicated in Table 6.1 were taken directly from the field records with the use of calibration curves as described previously, and represent the peak stress for each run. Stresses were not obtained from the surface transducer, however, because of damage during installation at the first site. In addition, no output was obtained from the lowest stress cell at Indiana 18. Since the cell proved undamaged on removal, and performed satisfactorily both in the laboratory and on subsequent tests at Indiana 26, the cause of failure was not definitely determined. A speculative explanation is that the soil was not properly compacted over the top of the transducer so that a void or bridging occurred, effectively eliminating any pressure from impinging upon the diaphragm.

Displacement records were produced on a digital computer, as previously described. A typical record of displacement is illustrated in Figure 4.16. Initially, the displacement used in studying the load-stress relationship was taken as the peak value from the list of discrete values. The values appeared to have an unreasonable amount of scatter, however, so values were later taken from the curves of displacement versus position of rear axle. In reviewing these curves, it was impossible to explain the comparatively large residual displacements, particularly since in general these were negative, except as errors resulting from drift in the analog integrator, oscillograph damping, and numeric integration. It was believed that more error was involved in accepting the raw displacement curves than in assuming no residual displacement. In all probability the correct displacement is somewhere between these extremes. In the absence of better information a residual displacement of zero was assumed. A new base line was drawn on each displacement curve to conform to this assumption, and the maximum displacement was measured from this base line to the peak of the curve. It is this value that is recorded for each run and each depth in Table 6.1. Displacements were not obtained for the three foot depth at very low speeds on McCormick Road and Indiana 26. Accelerations were so small in these cases that the oscillograph record could not be read accurately enough to obtain significant results. Several other observations were voided by drift of such magnitude that the record could not be interpreted. In addition, one amplifier failed during tests at Indiana 18 so that only two values of displacement are available for about half of the tests.

Values of surface displacement for the fastest runs at Indiana 18 and Indiana 26 show a significant variation from the overall pattern. This led to consideration of possible errors in the system at high frequency. Calculation of peak accelerations for these conditions suggest that the accelerometer range was exceeded. For example, a displacement of 0.030 inches in a time interval during which the truck moves ten feet at 60 miles per hour would require a peak acceleration of 0.15 g. Greater displacements and higher frequencies could possibly result in accelerations as much as 0.5 g. This acceleration is added to the constant 1 g of the earth's gravity. Recalling that the accelerometers were wired for a 1 g range, it is evident that the linear range of the accelerometer could have been exceeded enough to produce results which were seriously in error. This could be prevented easily in future applications by providing switch selected load resistors on the accelerometers for adjusting the range as necessary for varying magnitudes of displacement and vehicle speed. The questioned values were included in the analyses which follow, although one trial analysis was made excluding all runs in the 60 mile per hour range. No significant change was indicated by the omission -- slightly different coefficients in the regression equations were obtained, but the form was the same as that when all data were included.

Analysis and Discussion

Figures 6.1 through 6.9 show peak displacement versus velocity for the three pavements and Figures 6.10 through 6.12 show peak surface displacement versus offset. The first nine figures represent data grouped according to nominal lateral offsets of the truck of zero, six and 12 inches.

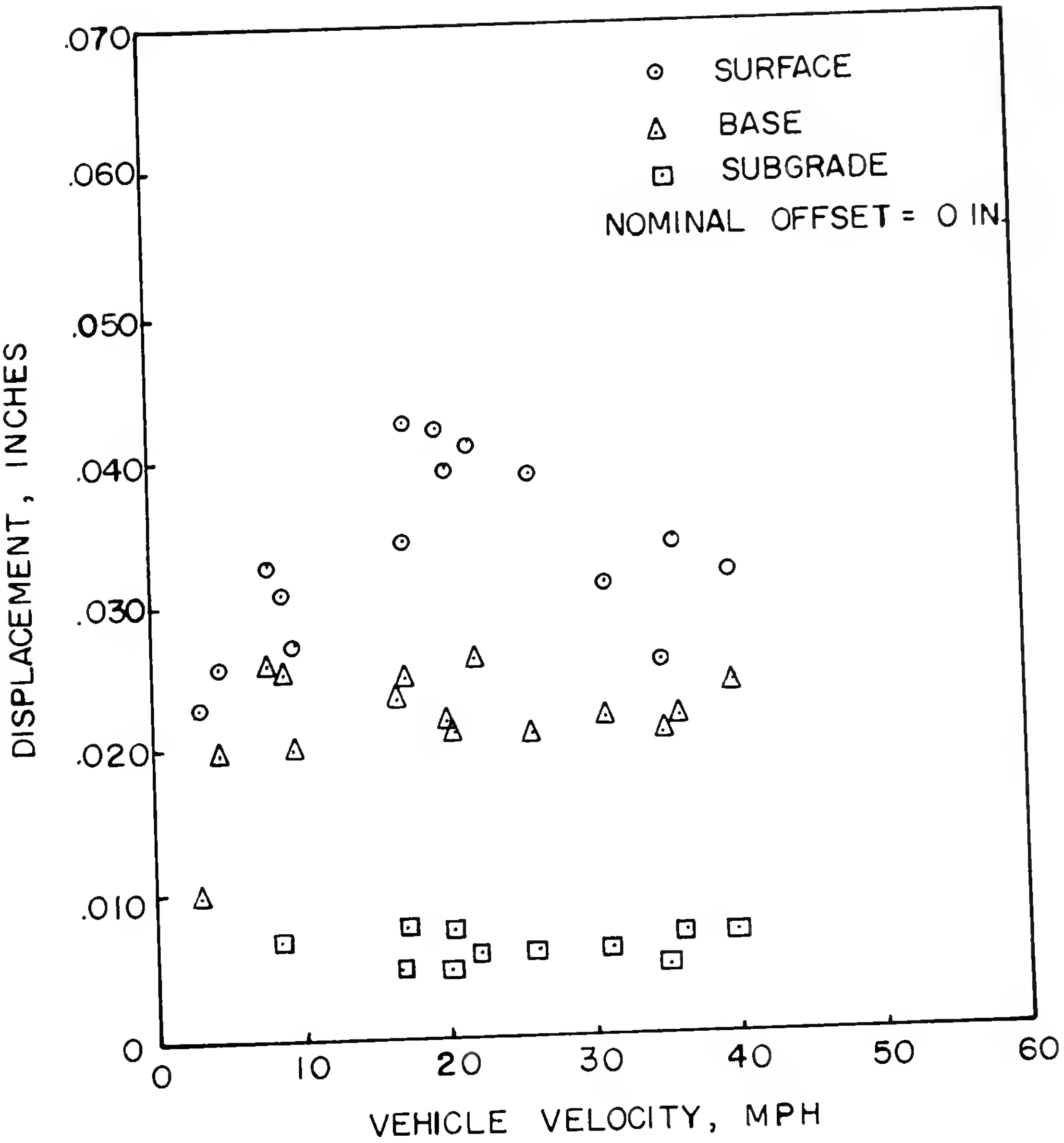


FIGURE 6.1. DISPLACEMENT VS. VEHICLE VELOCITY

McCORMICK ROAD

OFFSET = 0 IN.

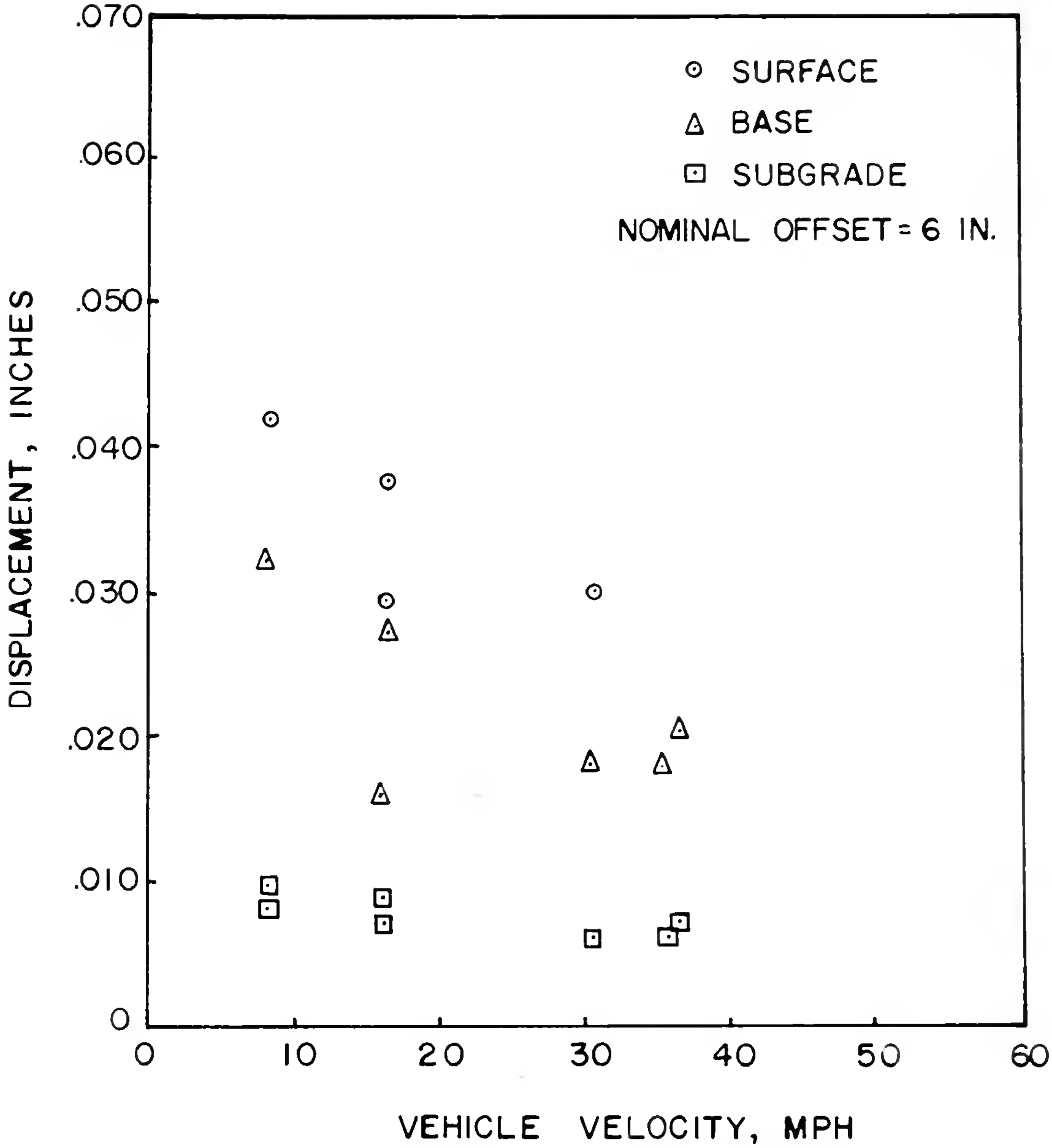


FIGURE 6.2. DISPLACEMENT VS. VEHICLE VELOCITY
McCORMICK ROAD
OFFSET = 6 IN.

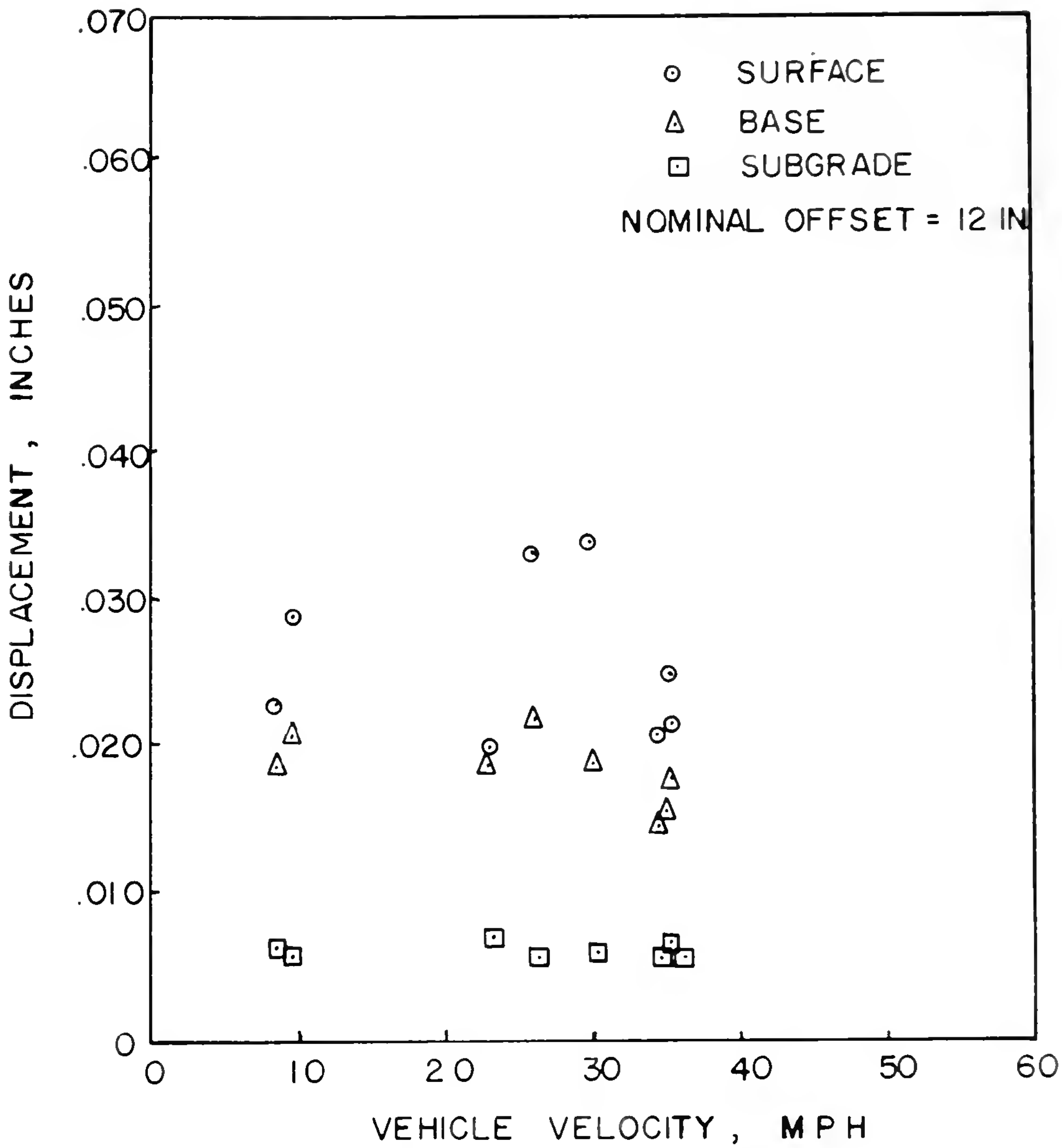


FIGURE 6.3. DISPLACEMENT VS. VEHICLE VELOCITY

McCORMICK ROAD

OFFSET = 12 IN.

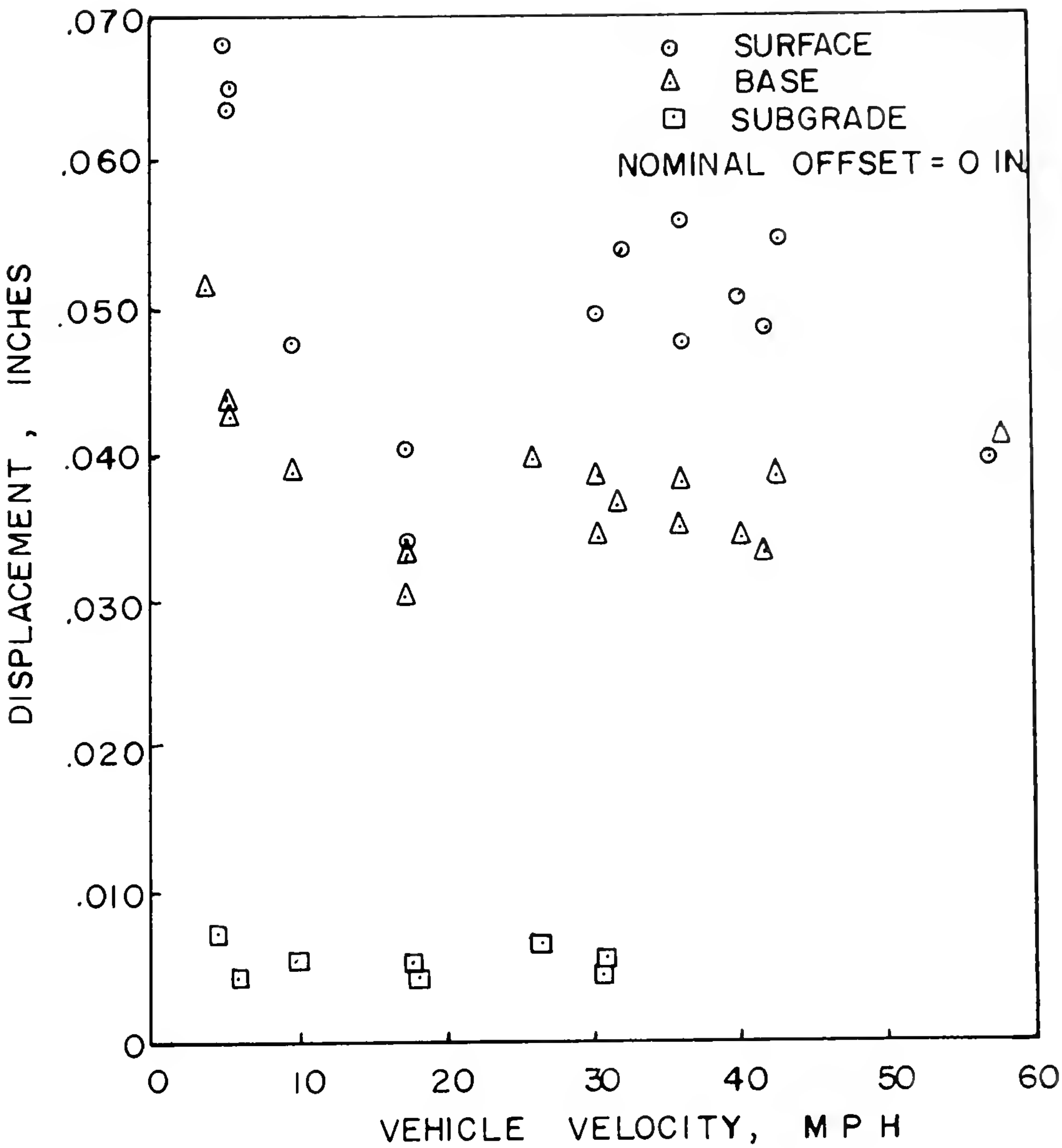


FIGURE 6.4. DISPLACEMENT VS. VEHICLE VELOCITY

INDIANA ROAD 18

OFFSET = 0 IN.

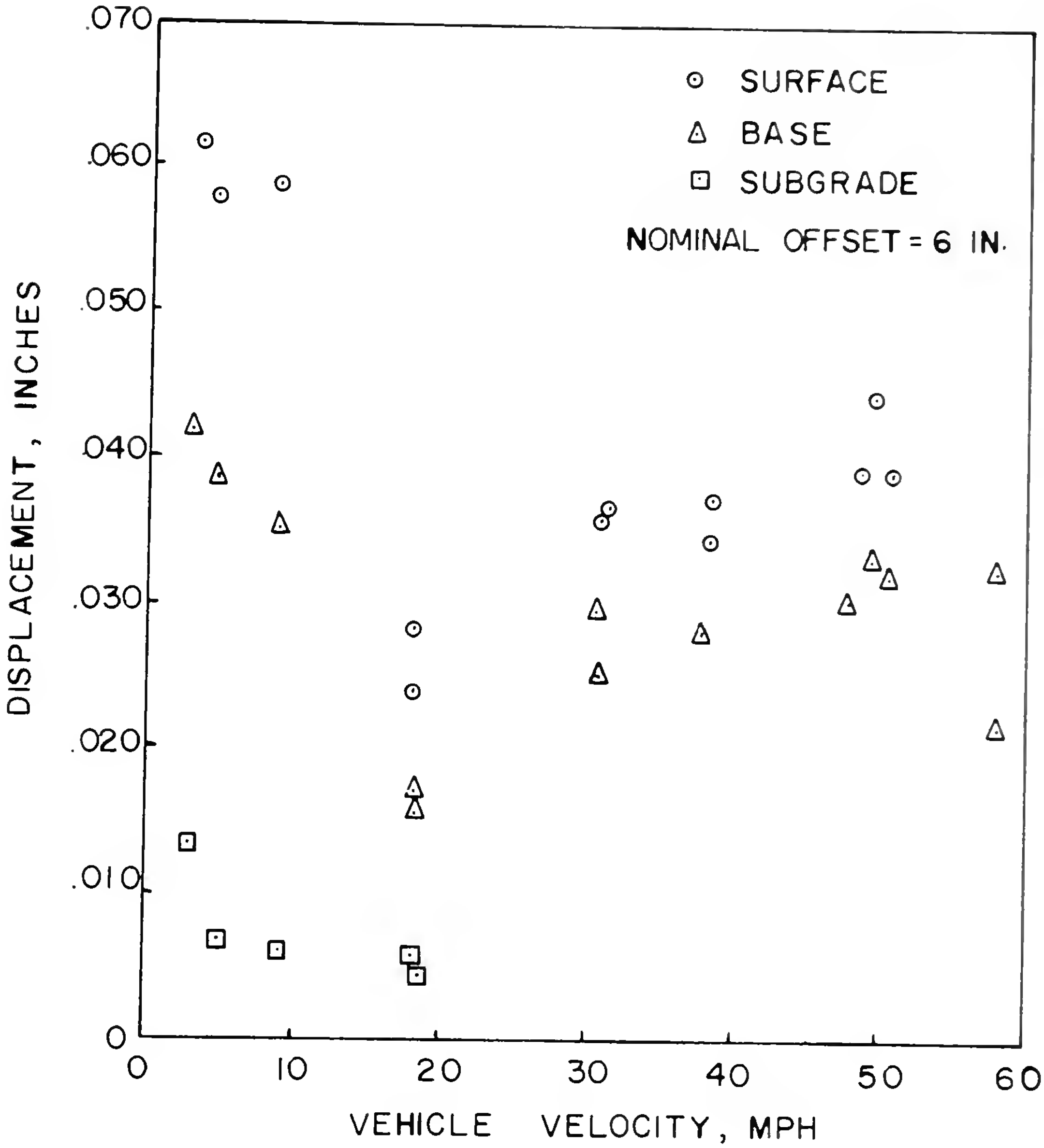


FIGURE 6.5. DISPLACEMENT VS. VEHICLE VELOCITY
INDIANA ROAD 18
OFFSET = 6 IN.

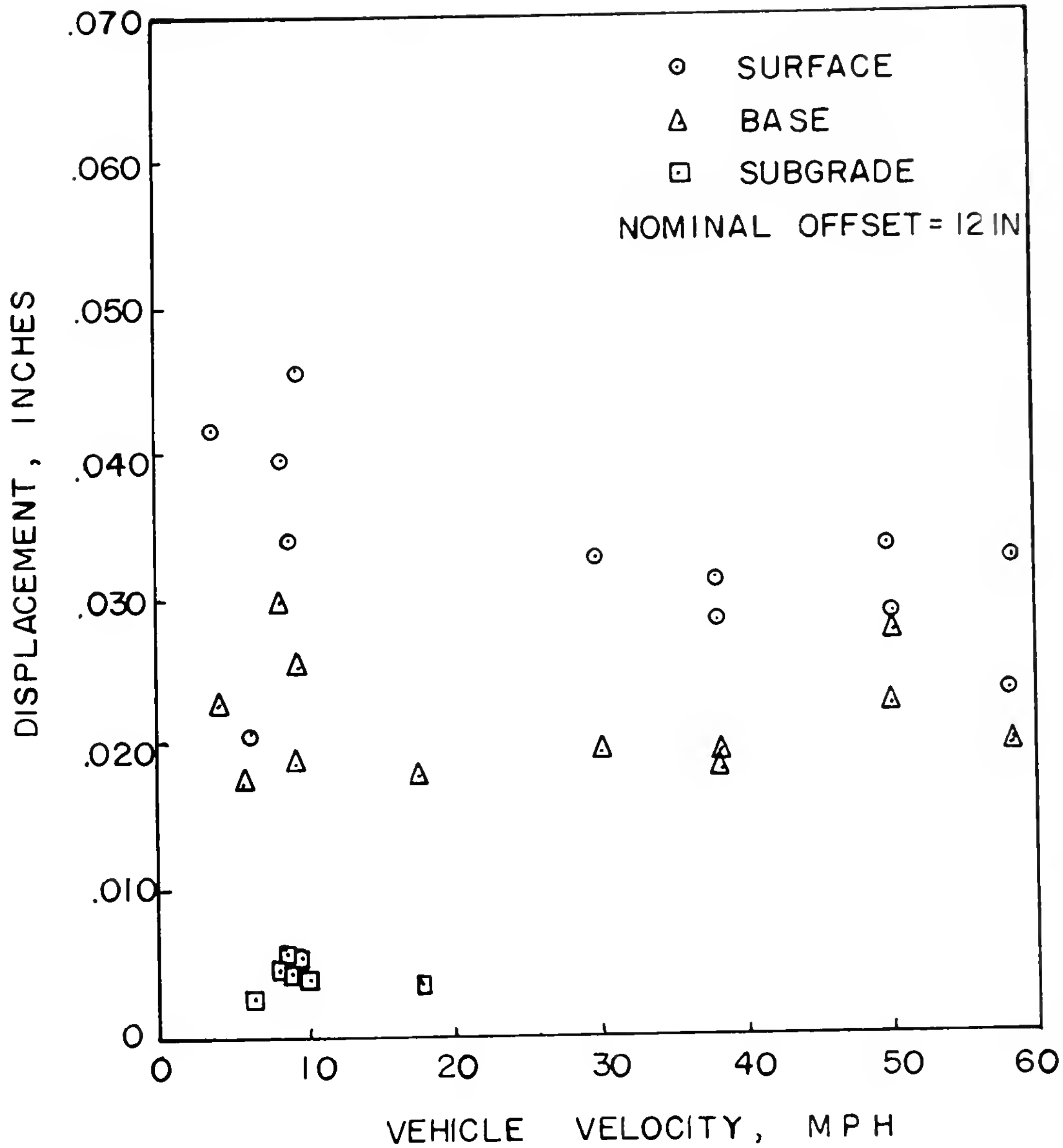


FIGURE 6.6. DISPLACEMENT VS. VEHICLE VELOCITY

INDIANA ROAD 18

OFFSET = 12 IN.

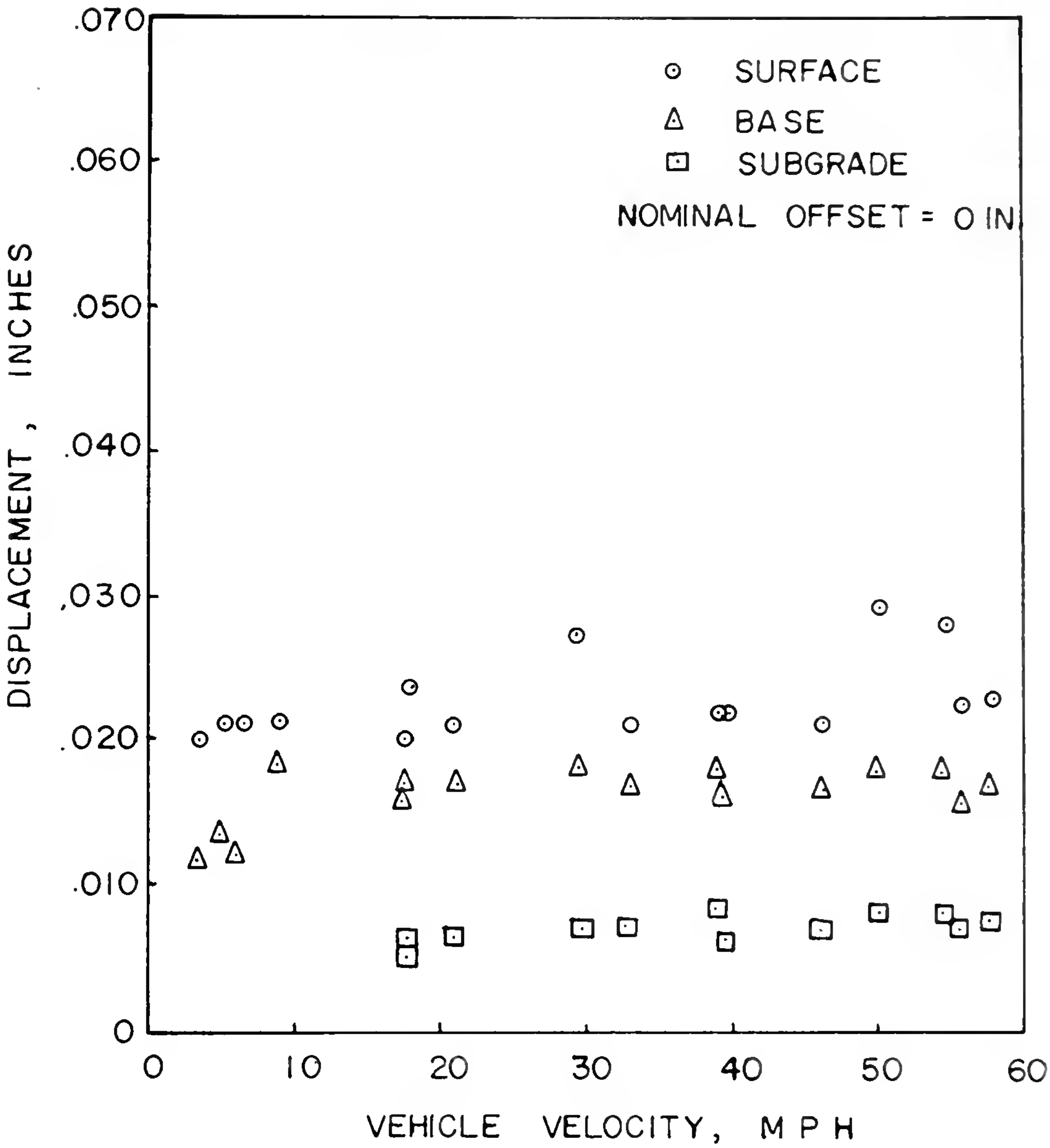


FIGURE 6.7. DISPLACEMENT VS. VEHICLE VELOCITY

INDIANA ROAD 26

OFFSET = 0 IN.

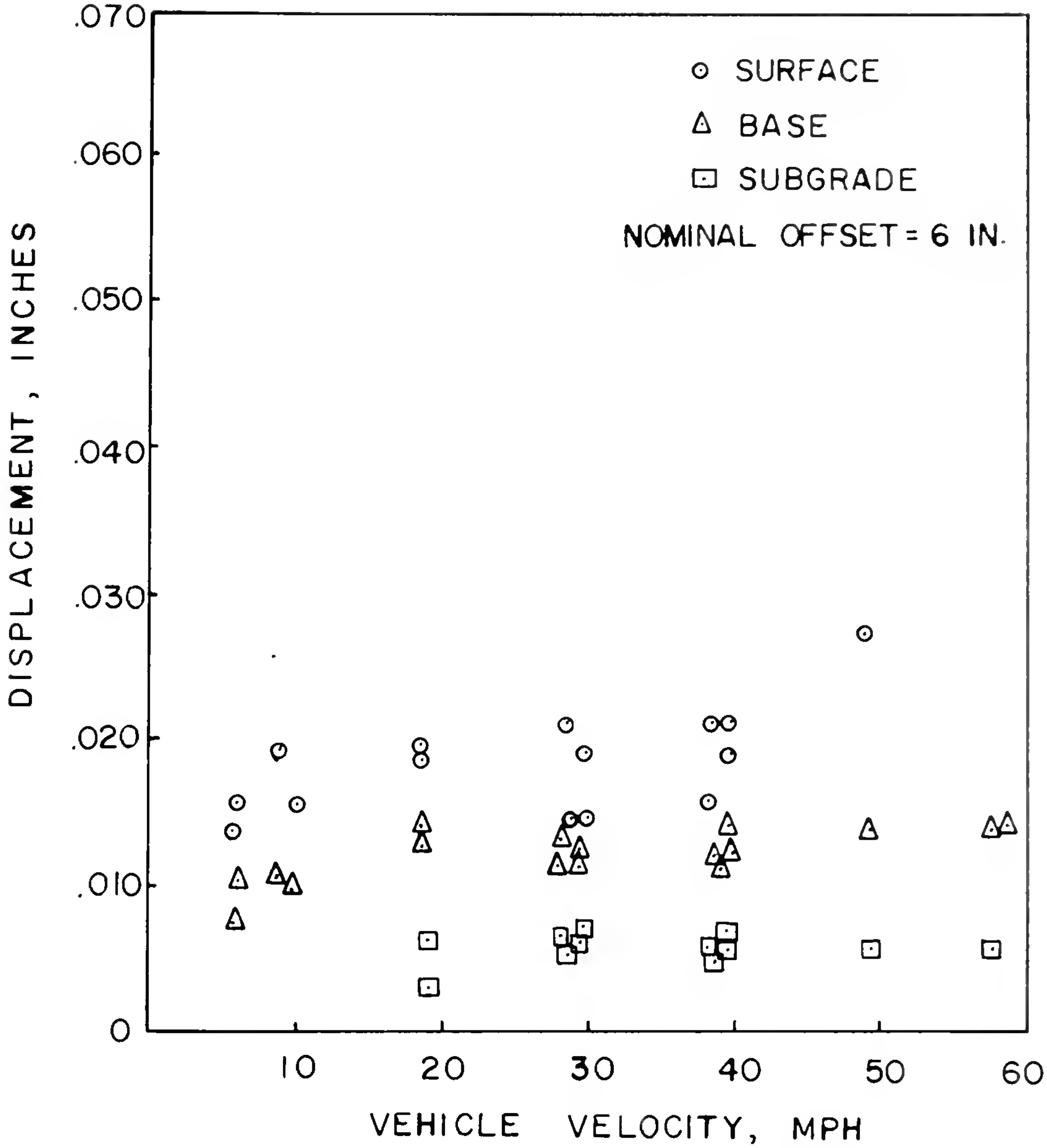


FIGURE 6.8. DISPLACEMENT VS. VEHICLE VELOCITY INDIANA ROAD 26 OFFSET = 6 IN.

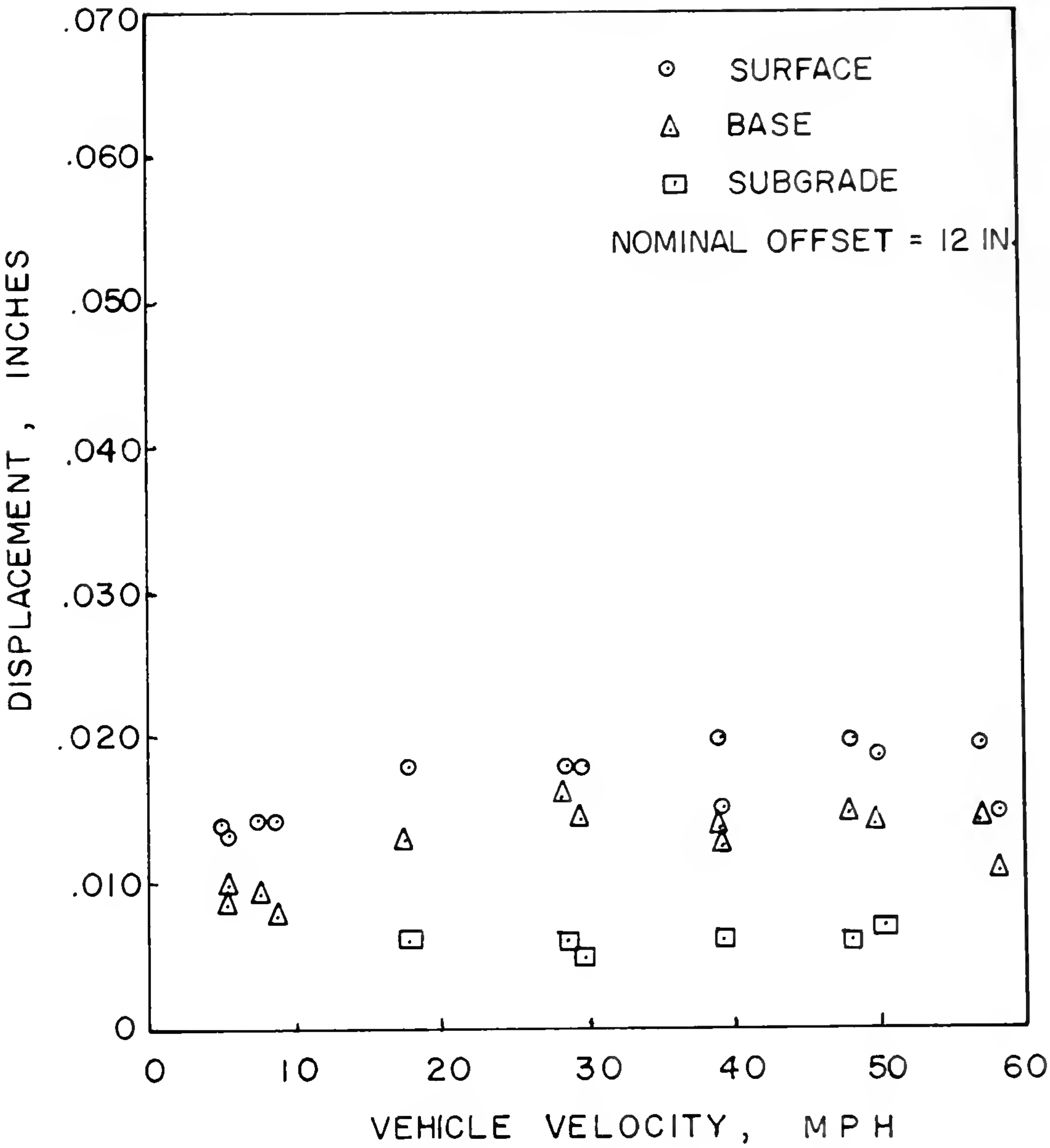


FIGURE 6.9. DISPLACEMENT VS. VEHICLE VELOCITY

INDIANA ROAD 26

OFFSET = 12 IN.

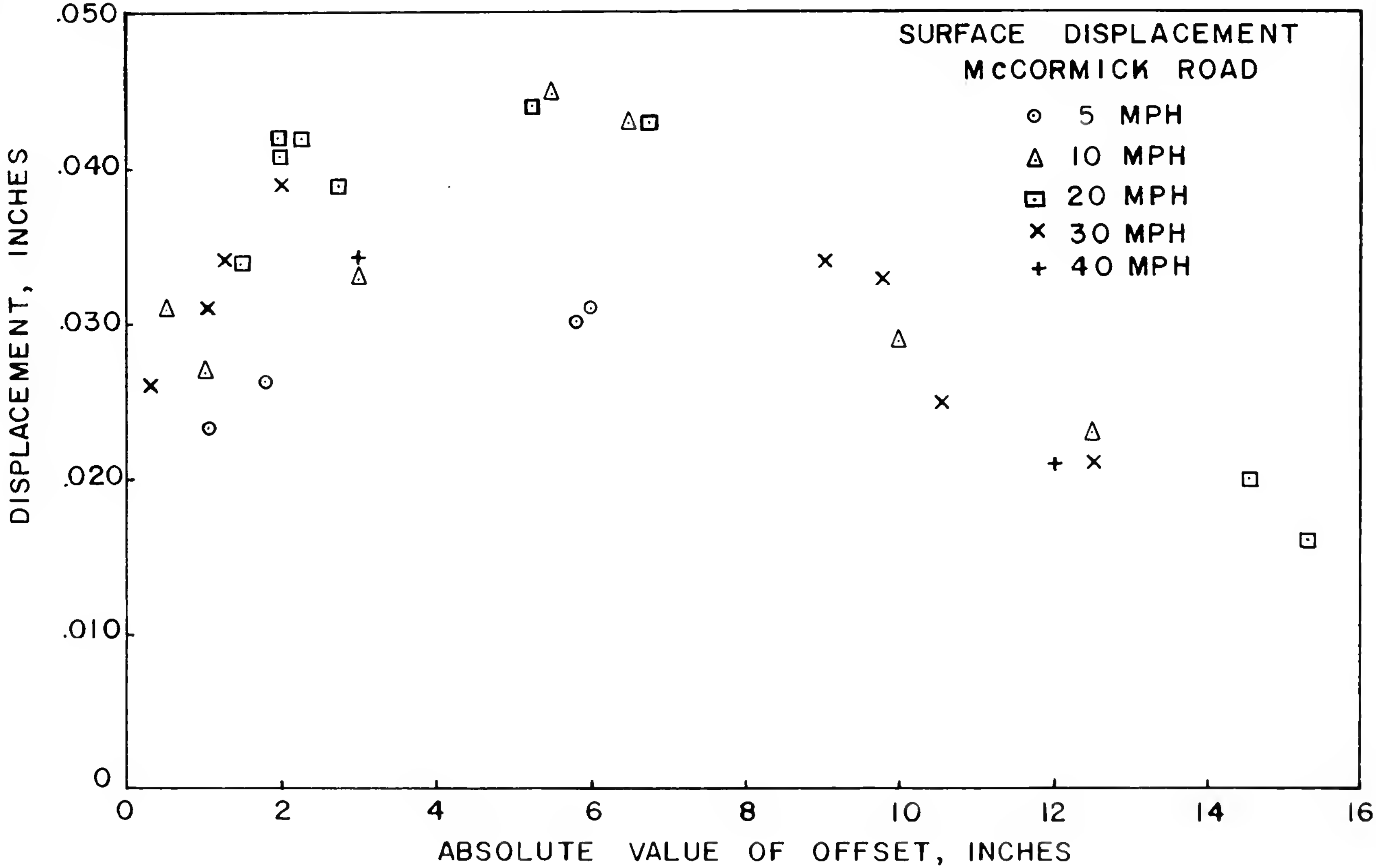


FIGURE 6.10. DISPLACEMENT VS. OFFSET, McCORMICK ROAD

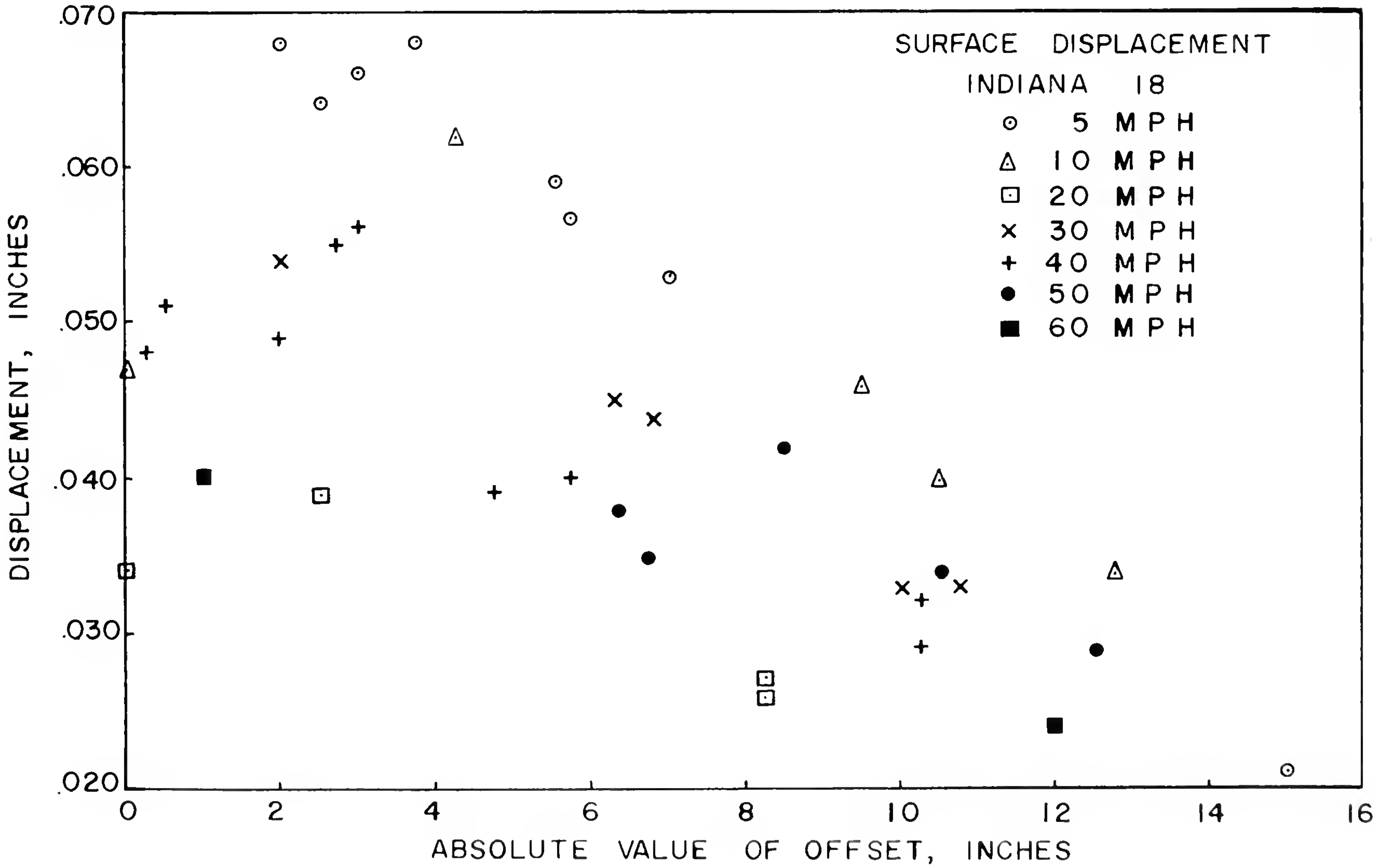


FIGURE 6.11. DISPLACEMENT VS. OFFSET, INDIANA ROAD 18

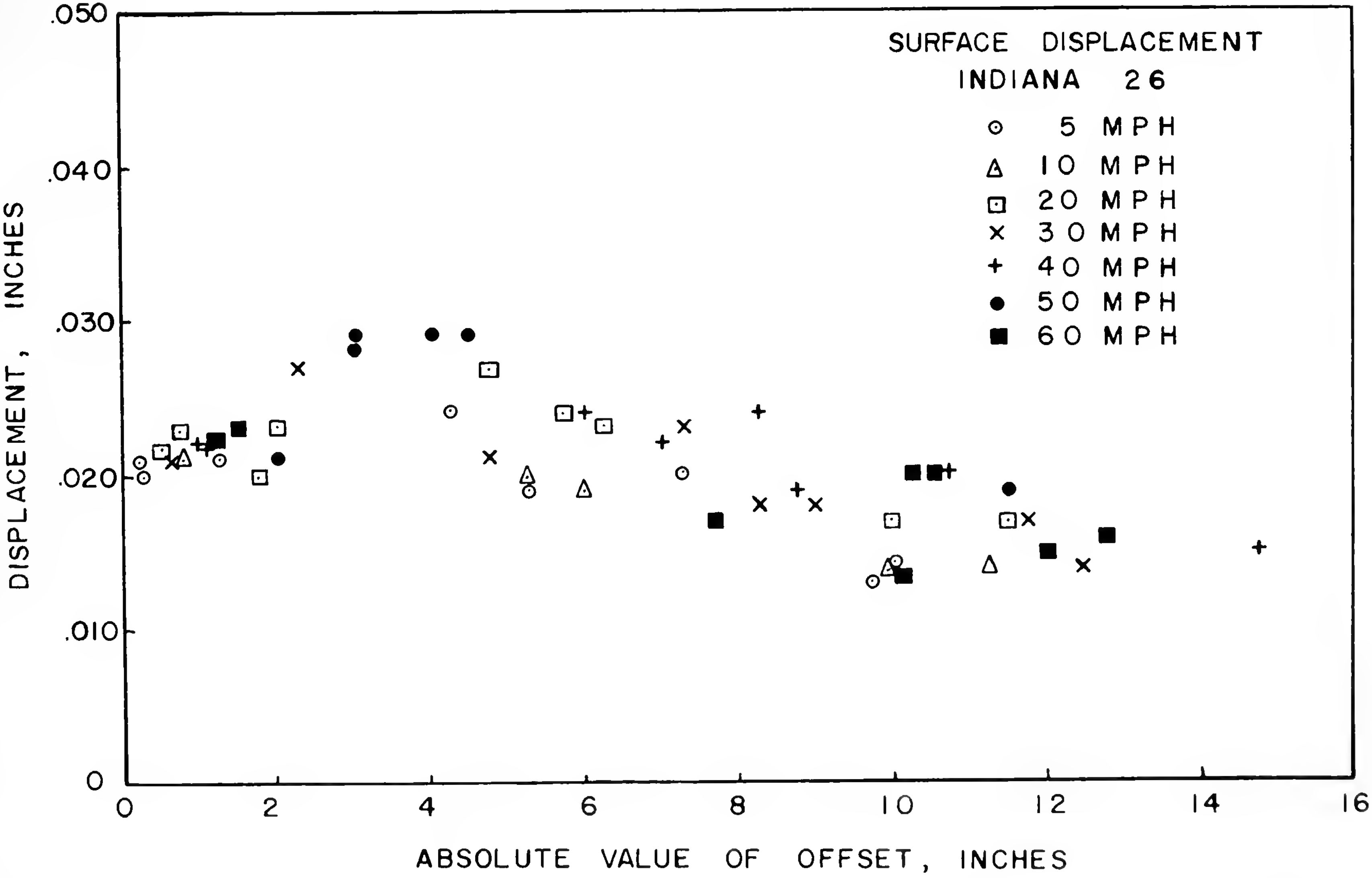


FIGURE 6.12. DISPLACEMENT VS. OFFSET, INDIANA ROAD 26

In the latter three figures, the points are identified according to the nominal velocity of the truck. Figures 6.13 and 6.14 present plots of peak stress versus vehicle velocity and offset, respectively, for the base-subgrade interface. These plots are identified according to pavement.

It is readily seen from these figures that the effect of various variables and parameters are so intermingled as to preclude the drawing of curves through the data. However, trends can be readily visualized from plots of any two of the variables.

The most striking relationship in the scatter diagrams is the pronounced dependence of both displacement and stress on the lateral position of the vehicle, or offset. In general, the maximum surface displacement occurred at an offset of three to five inches. This corresponds to a point in the contact area of one tire, approximately beneath the inner (towards the center line of the dual wheel) wall of the tire. From Figures 6.10 through 6.12 it can also be seen that maximum displacement is on the order of 50 percent greater than displacement at the center line of the duals. The observed variation with respect to offset is confirmed by equations fitted to the data, as described below. This relationship has important consequences in the interpretation of displacement data in general, and especially that obtained with the Benkelman Beam. As was pointed out previously, the Benkelman Beam can be used only outside the tire contact area. The usual position is between the two tires of a dual wheel. Clearly, this can result in significant errors if the observed displacement is considered to be maximum, particularly in connection with theoretical analyses.

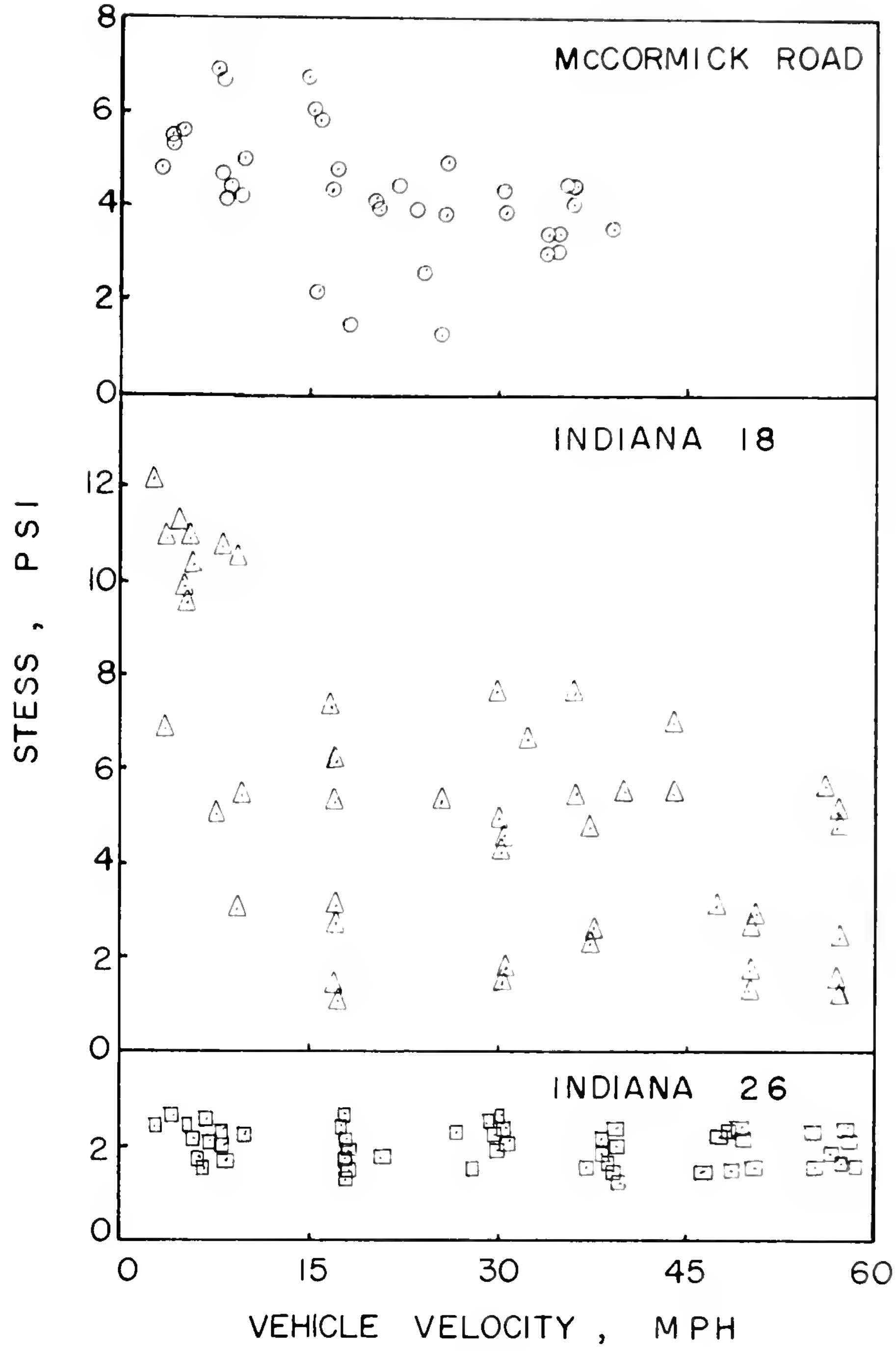


FIGURE 6.13. STRESS VS VEHICLE VELOCITY

BASE-SUBGRADE INTERFACE

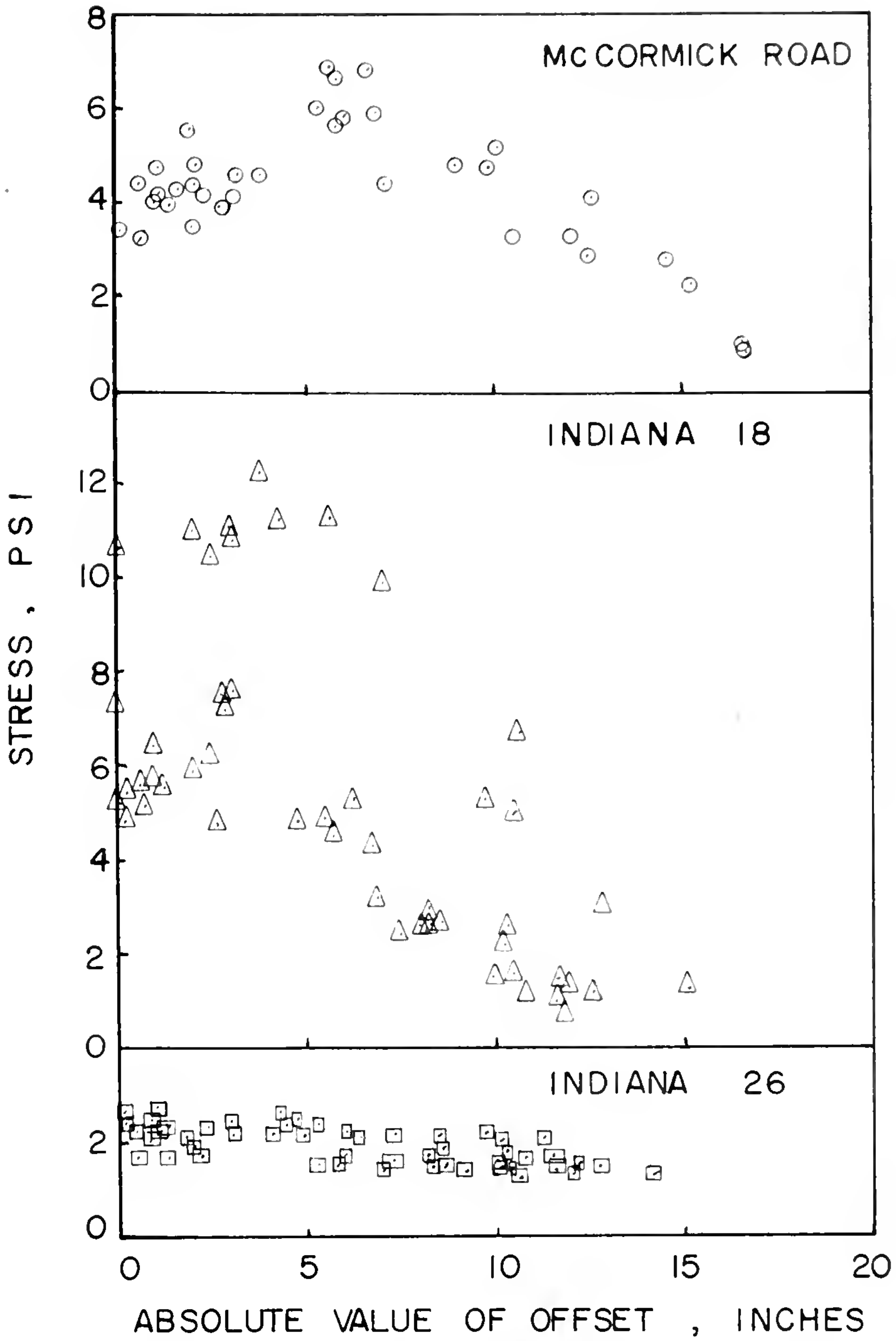


FIGURE 6.14. STRESS VS OFFSET BASE-SUBGRADE INTERFACE

There appears from Figures 6.1 through 6.9, and 6.13 to be a significant, though somewhat inconsistent, non-linear trend of displacement and stress with vehicle velocity. It is of interest to note that the greatest variation with velocity occurred in the data from Indiana 18 which was evidently the weakest of the three pavements. This site had the greatest displacements and stresses of the three tested. Figures 6.4, 6.5 and 6.6 indicate a distinct minimum displacement in the neighborhood of 20 to 30 mph and a rather large increase in displacement with velocity at higher vehicle speeds. Stresses, as seen in Figure 6.13, show a consistent reduction with increase in velocity.

While generalizations can be pictured from the scatter diagrams, as suggested above, some more definite measure of the effects of vehicle velocity on load and pavement response and their interaction was desired.

Mathematical Models

The data were analyzed by fitting displacement and stress data to the other measured variables using statistical techniques for the purpose of obtaining predicted displacement and stress values for given conditions of load, velocity and offset. The predicted values were then used for interpreting the pavement and subgrade response in terms of load and velocity.

The first step in obtaining a statistical fit was to determine a suitable mathematical model. A review of the scatter diagrams of Figures 6.1 through 6.14 provided some suggestions as to form. By combining these concepts with others suggested by theoretical solutions of stress and displacement for static loads, Model 1.1 in Table 6.2 was obtained. In

TABLE 6.2

MATHEMATICAL MODELS*

No.	Equation
1.1	$Y = A_0 + A_1P + A_2PV + A_3PV^2$ in which $P = P_0 \exp\left[-B_1 (\frac{R_0}{a})^2 - B_2 (\frac{Z}{s})^2\right] + P_1 \exp\left[-B_1 (\frac{R_1}{a})^2 - B_2(\frac{Z}{s})^2\right]$
1.2	$Y = A_0 + A_1P + A_2PV + A_3PV^2$ in which $P = P_0 \exp\left[-B_1 (\frac{R_0}{a})^2\right] + P_1 \exp\left[-B_1 (\frac{R_1}{a})^2\right]$
2.1	$Y = A_0P^{A_1}V^{A_2} \exp\left\{A_3 \left[(\frac{O}{a})^2 + (\frac{Z}{s})^2\right]\right\}$ in which $P = P_0 + P_1$
3.1	$Y = A_0P_0^{A_1} P_1^{A_2} V^{A_3} \exp\left\{A_4 \left[(\frac{R_0}{a})^2 + (\frac{Z}{s})^2\right]\right\} \exp\left\{A_5 \left[(\frac{R_1}{a})^2 + (\frac{Z}{s})^2\right]\right\}$
4.2	$Y = A_0 + A_1Q + A_2QV + A_3QV^2$ in which $Q = \exp\left[-B_1 (\frac{R_0}{a})^2\right] + \exp\left[-B_1 (\frac{R_1}{a})^2\right]$

* See Figure 16.3 for definition of terms

an attempt to make more specific statistical inferences, Models 1.2, 2.1, 3.1 and 4.2 in Table 6.2 were also evaluated.

Model 1.1 states that the dependent variable (either stress or displacement) at a point is proportional to the loads induced by both tires, P_0 and P_1, but that the stress (or displacement) contributed by each tire is in proportion to its distance from the point. The terminology used in the models of Table 6.2 is illustrated in Figure 6.15. The exponential term involves the second power of distance to account for the "hump" indicated in displacement versus offset. Since this hump is pronounced at the surface, but slight at the base-subgrade interface, and disappears deeper in the subgrade, it is evident that depth is as significant as horizontal offset, as would be expected. For the model to fit data for all depths as well as offsets, therefore, it was necessary to include both terms in the exponential factor. The variables representing horizontal offset, R_0 and R_1 in Model 1.1, were obtained respectively by subtracting from and adding to the offset, O, which was measured in the field, a constant, C. The value of C in each case was determined by inspection of the corresponding scatter diagram of displacement (or stress) versus offset. The distance parameters so calculated produce a hump in the model surface corresponding in offset to the location of the hump in the scatter diagrams for Figures 6.10 through 6.12 and 6.14. The final variable in Model 1.1 is vehicle velocity, V. Velocity is included to reflect the general trend of a non-linear dependence of stress and displacement on velocity as indicated by scatter diagrams of Figure 6.1 through 6.9 and 6.13. The constants "a" and "s" in the exponentials are not strictly necessary since the

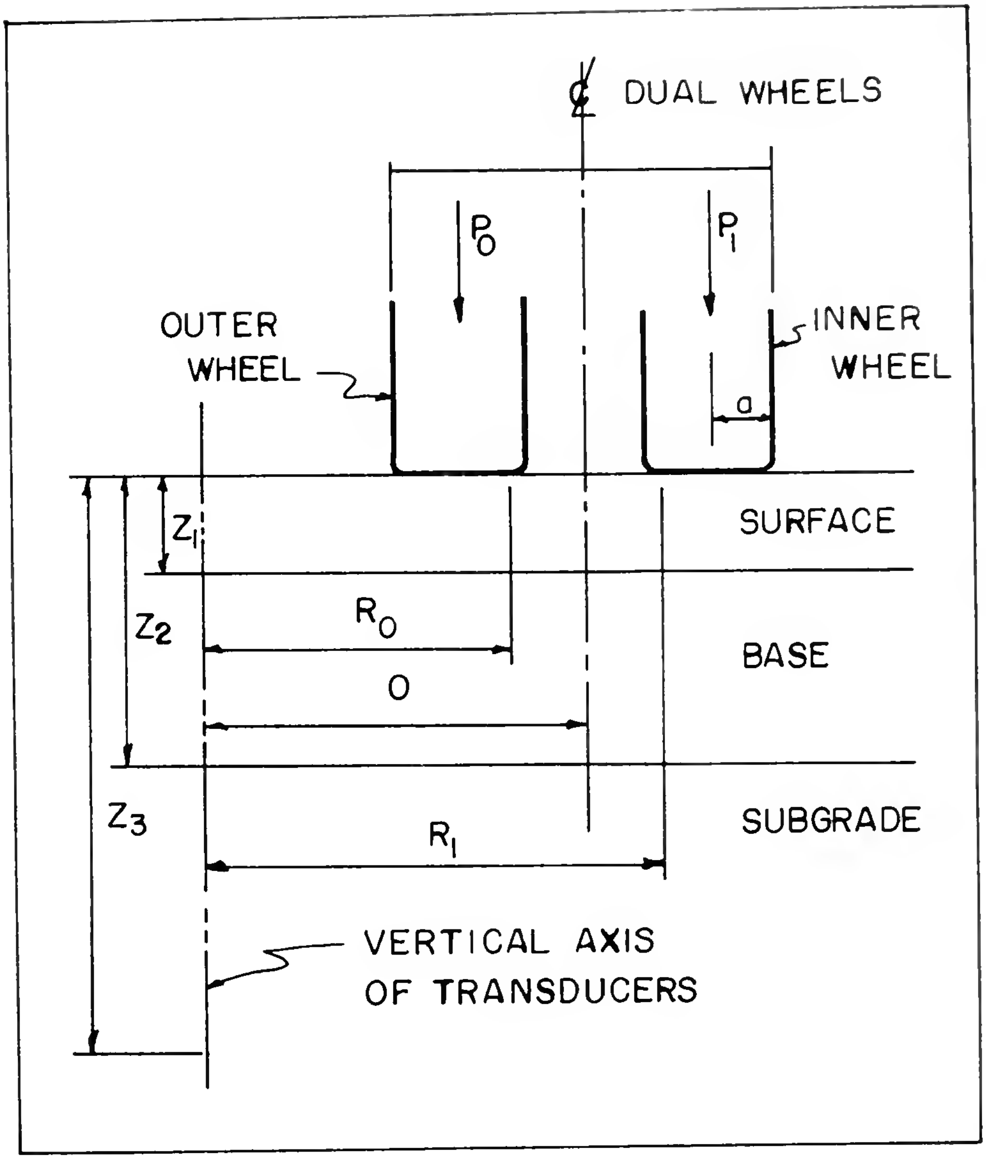


FIGURE 6.15. TERMINOLOGY OF MATHEMATICAL MODELS

coefficients B_1 and B_2 could include the effect of those factors. However, as will be seen subsequently, the evaluation of B_1 and B_2 was not a simple and explicit procedure, and by taking "a" to be one-half the minor axis of a single tire contact area, and "s" to be the overall width of the contact area, the exponents were made dimensionless and therefore general. Reasonably good fits to the observed data were obtained with these values of the exponents without solving explicitly for B_1 and B_2.

While Model 1.1 was developed largely on the basis of logic, it was not entirely satisfactory from a statistical point of view. As can be seen from Table 6.2, the equation is non-linear in the variable coefficients B_1 and B_2. With currently available techniques it was not possible to obtain a regression of this model on the data because of the non-linearity. In an attempt to satisfy the requirements of linearity, Model 2.1 was developed. This equation is linearized simply by a logarithmic transformation. Taking the natural logarithm of both sides of the original equation results in

$$\ln Y = A_0' + A_1 \ln P + A_2 \ln V + A_3 \left[\left(\frac{0}{a}\right)^2 + \left(\frac{Z}{s}\right)^2 \right] \qquad 6.1$$

in which the symbol ln represents logarithm to the base e and the remaining symbols are as previously defined.

A regression analysis on this model indicated an extremely poor fit to the data on the basis of the multiple regression coefficient, R^2, as a criterion. From Table 6.3 it is seen that the R^2 values for this model were only .23, .28, and .20 for McCormick Road, Indiana 18 and Indiana 26 respectively. That is to say, for example, the model explains only 23 percent of the variation in displacement about the mean displacement for

TABLE 6.3

SUMMARY OF MULTIPLE REGRESSION COEFFICIENTS (R^2)

Model	1.1				1.2		4.2			2.1	3.1
Site \ B	2	1.5	1	.5	1	.9	1	.9	.8		
<u>Displacement</u>											
McCormick Road	.45	.53	.64	.66						.23	.33
Surface					.42	.43		.43			
Base					.29	.30		.28			
Subgrade					.15	.15		.15			
Indiana Road 18	.52	.57	.62	.61						.28	.43
Surface					.75	.75	.77	.76	.76		
Base					.77	.77	.79	.80	.80		
Subgrade					.30	.57	.57	.57	.58		
Indiana Road 26	.41	.57	.65	.62						.20	.47
Surface					.54	.53	.53	.53			
Base					.45	.46	.46	.46			
Subgrade					.48	.48	.48	.47			
<u>Stress</u>											
McCormick Road					.59		.61				
Indiana Road 18					.81		.81				
Indiana Road 26					.38		.36				

NOTES: Models 1.1, 2.1 and 3.1 include independent variables of tire loads, offset, depth and vehicle velocity.
Model 1.2 includes independent variables of tire loads, offset and vehicle velocity.
Model 4.2 includes independent variables of offset and vehicle velocity.

$B = B_1 = B_2$

the McCormick Road data. It is not possible to determine from the regression analysis whether the unexplained variation is error in measurement, dependent on other variables not measured, dependent on some other function of the measured variables, or some combination of these factors.

Model 3.1 in Table 6.2 is a modification of Model 2.1 to account for the separate wheel loads and corresponding offsets. The linear transformation of this equation is

$$\ln Y = A_0' + A_1 \ln P_0 + A_2 \ln P_1 + A_3 \ln V + A_4 \left[\left(\frac{R_0}{a}\right)^2 + \left(\frac{Z}{s}\right)^2 \right] + A_5 \left[\left(\frac{R_1}{a}\right)^2 + \left(\frac{Z}{s}\right)^2 \right]. \qquad 6.2$$

On the basis of the R^2 values obtained for this equation, as shown in Table 6.3, the fit is improved somewhat over that of Model 2.1 by the modification. Even this performance, however, cannot be considered good.

The last equation in Table 6.2, Model 4.2, was used after further work had been done with Models 1.1 and 1.2 to aid in evaluating the significance of the load variation. Model 4.2 is similar to Model 1.2 but differs in that the loads P_0 and P_1 are eliminated as factors in Model 4.2. Models 1.2 and 4.2 are simplifications of Model 1.1 and were fitted to data from only one depth at a time in an attempt to improve the performance of the logical model.

Because the simple linear models resulted in such poor fits, they were considered of no direct usefulness in the analysis. It was necessary therefore, to attempt some analysis of Model 1.1 in spite of the difficulties imposed by the non-linear coefficients. The most promising approach seemed to be to assume values of B_1 and B_2 and to evaluate the coefficients

A_0, A_1, A_2 and A_3. A cut and try procedure was adopted to evaluate the effect of B_1 and B_2, beginning with the values

$$B_1 = B_2 = 1 \qquad 6.3$$

and working with displacement data from the three test pavements independently. It can be surmised that these values are not optimum, since the R^2 for the three sites at McCormick Road, Indiana 18 and Indiana 26 were only .64, .62 and .65 respectively. However, the solution gives no indication of how B_1 and B_2 should be varied.

The next step taken to improve the prediction was to eliminate the variable factor $B_2\ (\frac{Z}{s})^2$ from the equation of Model 1.1, resulting in Model 1.2, and obtain three separate solutions for each site - that is, a regression equation for data from each depth. This gave an improved fit, at least for the upper levels, for the data from Indiana 18, but was no improvement at McCormick Road and Indiana 26.

Analyses were then run with Model 1.2 using 0.9 for B_1. Preliminary studies of the form of the term

$$Y_1 = \exp\left[-B_1\ (\frac{R_0}{a})^2\right] + \exp\left[-B_1\ (\frac{R_1}{a})^2\right] \qquad 6.4$$

had indicated that values of B_1 much less than 1 would damp out the hump necessary to fit the variation of stress and displacement with offset while values of B_1 greater than 1 would exaggerate the hump out of proportion to the observed data. However, the fit appeared comparatively insensitive to small variations in B_1. A summary of the R^2 values obtained is given in Table 6.3. A comparison of R^2 values for Model 1.2 for the different data sets indicates almost precisely the same performance for

values of $B_1 = 1$ and $B_1 = 0.9$. A similar result is seen for Model 4.2 in which values of 1, 0.9 and 0.8 were used for B_1.

These results indicated no advantage in using separate prediction equations for the different levels at which displacements were observed. It was therefore decided to study further the effect of the non-linear coefficients in Model 1.1.

Since it was not possible to determine the relationship between B_1 and B_2, it was assumed for simplicity that

$$B_1 = B_2 = B \qquad 6.5$$

and regressions were run on Model 1.1 using values of $B = 2$, 1.5 and 0.5. As seen in the summary of Table 6.2, values of B greater than 1 resulted in much poorer performance of the model, while a value of less than 1 did not give consistently better results. Therefore, the value of $B = 1$ was assumed for the prediction equations. The resulting coefficients A_0, A_1, A_2 and A_3 in Model 1.1 are presented in Table 6.4

TABLE 6.4

COEFFICIENTS OF THE INDEPENDENT VARIABLES OF REGRESSION

	A_0	A_1	A_2	A_3
Displacement (Model 1.1)				
McCormick Road	.01200	.002284	.0006198	-.00001177
Indiana Road 18	.01723	.01207	-.0002460	.000003236
Indiana Road 26	.01042	.002778	.0001038	-.000001543
Stress (Model 1.2)				
McCormick Road	3.172	.8080	-.01702	.00005905
Indiana Road 18	2.377	2.748	-.07826	.0007223
Indiana Road 26	1.732	.1684	-.003835	.00005178

Note: $B_1 = B_2 = 1$

On the basis of experience with Model 1.2 on displacement data the same model was applied to stresses. The coefficients obtained in these analyses are also shown in Table 6.4. The fit of these data proved better than that of displacement for Indiana 18, as indicated by the comparatively high value of R^2 shown in Table 6.3, and essentially the same as that of displacement for McCormick Road. The fit to the Indiana 26 data was not as good as for the other roads. This was attributed to the rather small amount of variation in stress indicated at this site. The stresses at all vehicle velocities and offsets were low, apparently due to the comparatively stiff pavement structure. The stress cell, with present associated equipment, does not have adequate resolution to distinguish variations in stress less than about one psi. Hence the measured variables could not be expected to explain much of the indicated variation.

Pavement Material Characteristics

At the time of removing transducers from the test sections, samples of the base course and subgrade materials were obtained for laboratory testing. In addition to the routine classification tests, triaxial tests were made on these materials in the Soils Laboratory. Four by eight inch cylinders were molded to the field conditions of moisture and density. Using a triaxial apparatus, the samples were subjected to a confining pressure of 20 psi and loaded at a rate of strain of 0.05 inches per minute to failure or until the limit of the equipment was reached.

Modulus of deformation was calculated for each of these materials from the laboratory curves as the ratio of deviator stress to strain at an arbitrary stress corresponding approximately to average vertical stress in the material as described above. For the base course materials, a stress level of 40 psi was used and for the subgrades, 5 psi. The resulting moduli are indicated in Table 6.5.

TABLE 6.5

SUMMARY OF LABORATORY TEST RESULTS

Site	Modulus of Deformation: Base Course (at σ = 40 psi) psi	Modulus of Deformation: Subgrade (at σ = 5 psi) psi
McCormick Road	3370	1330
Indiana Road 18	9150	1140
Indiana Road 26	16000	30

On the basis of the results of the field tests it appears probable that the modulus of deformation of a pavement subgrade is a complex function of material types and conditions as well as the rate of loading. It seems reasonable to surmise that if the appropriate function could be determined, and evaluated for different pavement materials, that a parameter involving ratios r_B and r_S, as a function of vehicle velocity, could be included in a logical model for pavement displacement. Such a model might take the form

$$Y = A_0 + A_1 \left(\frac{P}{D}\right) \qquad 6.5$$

where D is a term incorporating r_B and r_S (as a function of vehicle velocity), and P is as used in Model 1.1, Table 6.2.

In order to determine the function D, it would be necessary to run a series of tests similar to that conducted in this study, but including multiple installations in each pavement. This would make possible a statistical evaluation of D for each pavement by the application of Model 1.1. The resulting values of D could then be incorporated in equation 6.5 in order to evaluate the coefficients A_0 and A_1 which would be applicable to all pavements in the analysis. As an ultimate goal, the evaluation of

D from laboratory tests is suggested. If such an evaluation is made, it will provide, along with the appropriate coefficients in equation 6.5, a means of predicting the response of pavements to dynamic loads.

Effect of Load Variation

If it is assumed that the relation between r_B and r_S and velocity is not fundamental but results from the variation in displacement and stress from which they are calculated, it should be possible to explain the variation of the measured parameters on the basis of load changes. Hence, it was of primary interest in this study to determine the extent to which impact effects of high speed loads influenced pavement response. For this purpose, regressions of Model 4.2 in Table 6.2. were carried out on the displacement data from all three test pavements.

As seen from Table 6.3, the fit of Model 4.2 is substantially the same as that of Model 1.2 both for stress and displacement, suggesting that the load factor in Model 1.1 did not contribute significantly in explaining the variation. There is a possibility, however, that the magnitude of load as the wheel approaches the test point is more significant than the load just as the wheel passes over the point or that whether load is increasing or decreasing as the wheel approaches the observed point is significant. Further studies should be made of these points.

A further study of the question of dynamic load involved a consideration of the average load over a section of pavement. The values of load at closely spaced discrete points over a distance of approximately 40 feet was obtained as described in Part 5. The values were averaged for all runs, both for the inner and outer wheels separately and for the total

load. Plots of average total load versus vehicle velocity are shown in Figures 6.16 through 6.18. Only a very slight trend of load with vehicle velocity is evident from the data. This would not suggest that, on the average, variation of load as a function of vehicle speed would produce large variations in pavement response.

The above statement must be qualified on the basis of the single truck and load (18,000 lb axle), and the pavements studied. The significance of the effect of load variation with velocity for design and evaluation of flexible pavements, if verified on a variety of pavements, is apparent. Confirmation of this finding will require tests of a variety of vehicles at a wide range of vehicle loads for each vehicle.

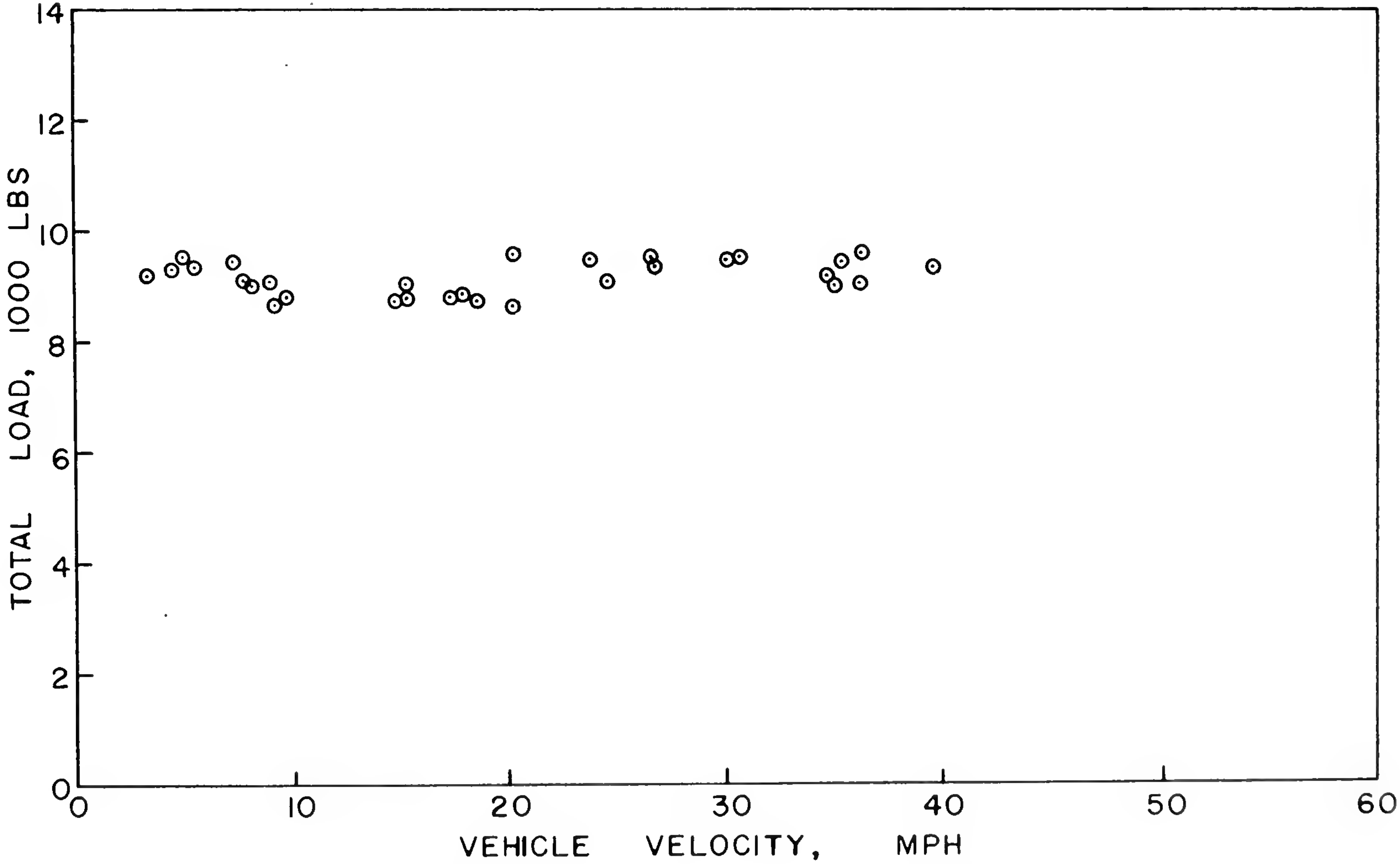


FIGURE 6.16. AVERAGE LOAD VS VEHICLE VELOCITY, McCORMICK ROAD

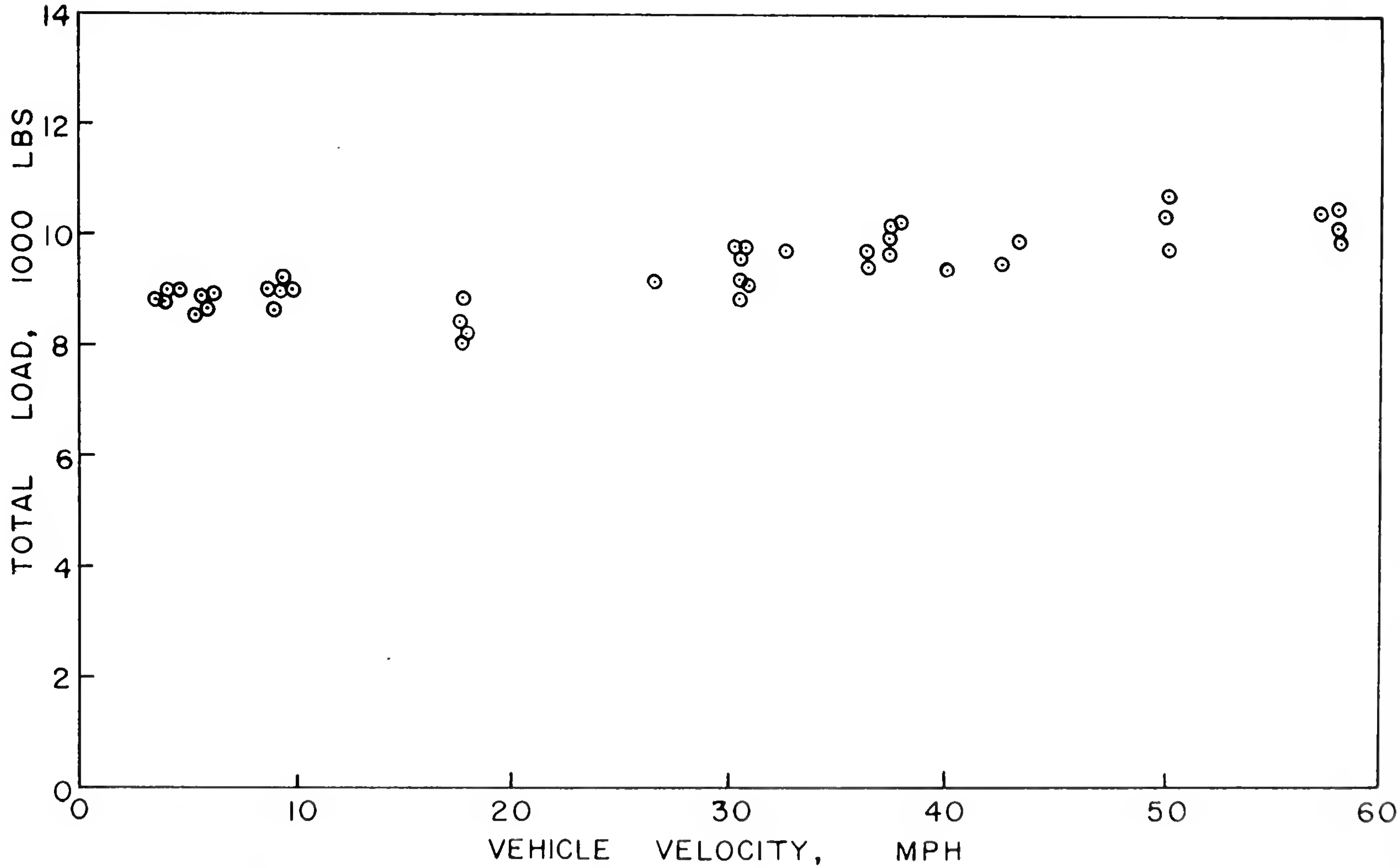


FIGURE 6.17 AVERAGE LOAD VS VEHICLE VELOCITY, INDIANA ROAD 18

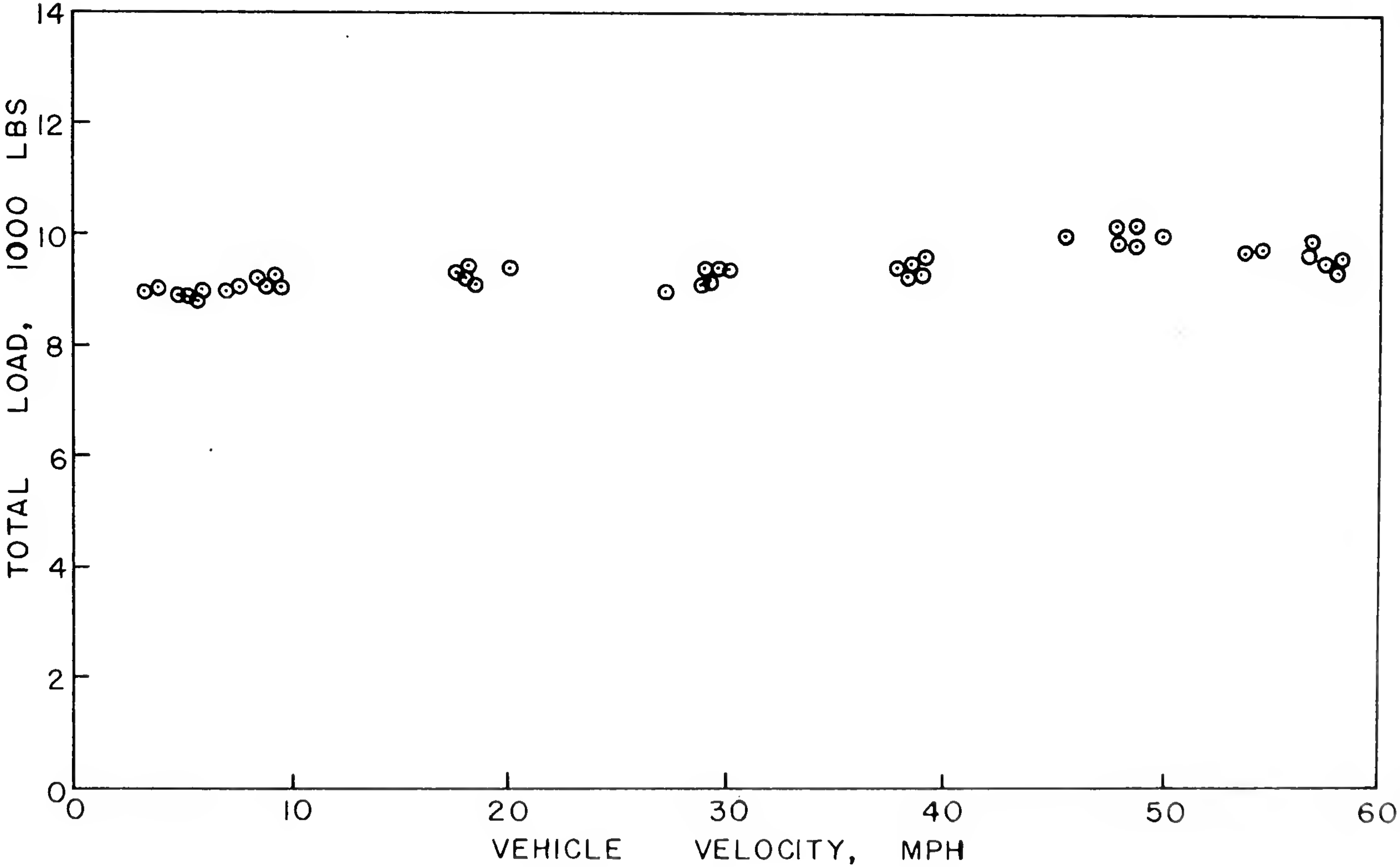


FIGURE 6.18. AVERAGE LOAD VS VEHICLE VELOCITY, INDIANA ROAD 26

7. SUMMARY AND CONCLUSION

Summary

This study was concerned with the development and application of equipment and techniques for the measurement of dynamic tire loads produced by heavy highway vehicles, and the dynamic response of flexible pavements to those loads. The effect of vehicle velocity on load and response was a major subject of the investigation. The study was planned to produce information about the variables entirely from field observations without applying laboratory test data to the interpretation of results.

Dynamic tire loads were measured by monitoring variation of tire inflation pressure. Dynamic calibration of the force-pressure ratio and mathematical techniques of accounting for the frequency dependence of that ratio were used. Records were obtained of tire force versus position on the highway for each tire of a dual wheel of a dump truck on three different pavements.

Vertical stresses and displacements at three levels within the pavement and subgrade were measured for the same pavements on which (and at the same time as) loads were measured. Stresses were measured with a flexible diaphragm type pressure cell. The displacement transducer was a servo-accelerometer. The acceleration signal was integrated once in the field using a specially built electronic analog device. The resulting oscillograph record of velocity of displacement was numerically integrated to obtain displacements.

Plots of stress and displacement are presented in which an important dependence on lateral position of the test vehicle is apparent. Trends of stress and displacement as a function of vehicle velocity are also indicated. Statistical curve fitting techniques were used to fit the data to a model which relates peak stress and displacement to instantaneous load at the test point, lateral position of the vehicle and vehicle velocity.

Using the fitted equations, predicted values of stress and displacement were obtained for particular values of load and vehicle position as a function of vehicle velocity. The ratios of average vertical stress to average vertical strain in the base course and in the subgrade were computed from predicted values of stress and displacement and the variation of these ratios with vehicle velocity is demonstrated. Certain results are presented which indicate that dynamic effects of load were unimportant with respect to pavement response in the case of the vehicle and pavements in this study.

Conclusions

The results of this study indicate the following:

1. It is feasible to measure dynamic tire forces of heavy vehicles by monitoring fluctuations of tire inflation pressure.
2. Dynamic calibration of force-pressure ratio is essential for valid interpretation of tire forces of heavy vehicles.
3. It is necessary to account for frequency dependence of the force-pressure ratio in evaluating tire forces from inflation pressure measurements.

4. Measurements of dynamic stresses within a flexible pavement subject to moving vehicle loads can be made with a flexible diaphragm type of cell.
5. The inertial system, employing an acceleration sensitive transducer, is feasible for measuring dynamic displacements of flexible pavements subjected to moving vehicle loads.
6. Displacements within the pavements studied varied greatly in the immediate vicinity of the dual wheel. Maximum displacement occurred under the tire walls and significantly smaller displacements occurred between the tires.
7. Displacements and stresses within the pavements studied varied with vehicle velocity as well as with pavement stiffness. Subgrade displacements did not show significant variation.
8. The effect of dynamic variations of load for the load and range of velocity covered in the study is minor in terms of its influence on pavement response.

Recommendations

On the basis of results obtained in this study, it is recommended that further study of the response of flexible pavements to load as a function of vehicle velocity be undertaken. Of major importance are, 1) verification that variation of load due to velocity is of minor importance and, 2) further study of the character of the stress-strain ratios of base and subgrade materials.

The accomplishment of the first of these will require testing of a wide range of pavements, including rougher surfaces than were included in

this study. The effects of different vehicles and different axle loads would also have to be determined in order to obtain conclusive evidence of the results indicated to date. A further study of the character of the stress-strain ratios will require multiple installations of stress and displacement measuring equipment in each of the pavements under study. This would make possible estimates of the variability of character of a single pavement and give reliable measures on which to base comparisons of different pavements.

It would be most desirable to obtain further laboratory tests on the materials from the pavements reported in this study. Of especial significance would be evaluations of modulus of deformation for these materials evaluated at differing rates of strain and at differing confining pressures. Such data would be of value in determining why the form of the stress-strain ratio as a function of vehicle velocity is different for different pavements. It also could contribute to the development of techniques for evaluating from laboratory tests a parameter of pavement response as a function of vehicle velocity.

Certain modifications in equipment are suggested by the experience of this study. The stress cells performed reasonably well though their sensitivity was marginal, especially for measurements deep in the subgrade. Sensitivity could be improved by the use of high quality, stable dc amplifiers and dc excitation to replace the ac equipment used in this study.

Further work on the displacement measuring equipment is essential for any extensive program. The manual digitalization and numeric integration of velocity of displacement records is too slow for making a large

number of observations, and limits the accuracy and precision of measurements. It appears justifiable to resume efforts to double integrate the accelerometer signal electronically.

Two possible approaches to this problem have been considered and are suggested here. The first method, using basically the same system as in this study, is to incorporate high quality dc data amplifiers for preamplification of the signal and solid state operational amplifiers for the integration. There is some evidence that these modifications could make the inertial system as originally conceived suitable for obtaining displacements directly in the field. As an alternate approach the second method would be to record the acceleration signal directly on magnetic tape and process it in a laboratory using a high quality electronic analog computer. This would produce no displacement data in the field so that degree of success of data collection would not be known until a later date. However, the transducer and techniques of operation proved quite reliable in the present study. In addition, field monitoring of the system could be easily provided with a meter or a single channel computer adequate for operational checks.

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